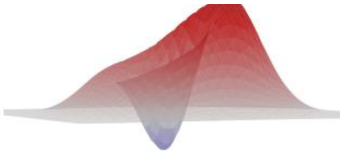

Non-linear quantum optics

Alexei Ourjoumtsev

Young Physics Teams
Collège de France

Valentin Magro (ex-PhD)
Julien Vaneecloo (ex-PhD)
Antoine Covolo (PhD)
Yi Li (PhD)
Mathieu Girard (PhD)
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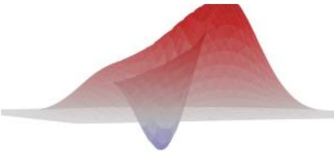
Pierre Treussart (PhD)
Sébastien Garcia (CNRS)

jeipcdf.cnrs.fr/quantum-photonics/



Non-linear quantum optics

Exercise:

- Undergraduate-level calculation
- Solution available (alexei.ourjoumtsev @college-de-france.fr)

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Non-linear quantum optics

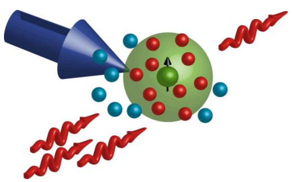
DiVincenzo's criteria
Isolate physical qubit
Initialize
Single-qubit gates
Multi-qubit gates
Store
Detect
Transmit

Non-linear quantum optics

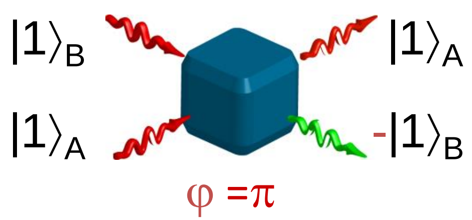
DiVincenzo’s criteria	Optical photons:
Isolate physical qubit	Hard
Initialize	Easy
Single-qubit gates	Easy
Multi-qubit gates	Hard
Store	Hard
Detect	Easy
Transmit	Easy

Need photon-photon interactions:

Non-linear absorption:



Non-linear dispersion:



Non-linear quantum optics

DiVincenzo's criteria	Optical photons:
Isolate physical qubit	Hard
Initialize	Easy
Single-qubit gates	Easy
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Transmit	Easy

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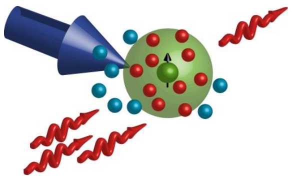
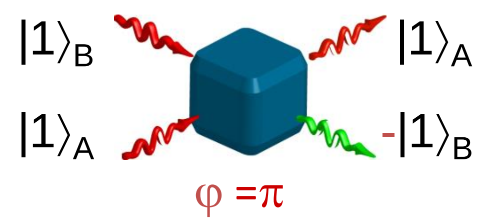


Non-linear dispersion:



Ask Jean

Non-linear quantum optics

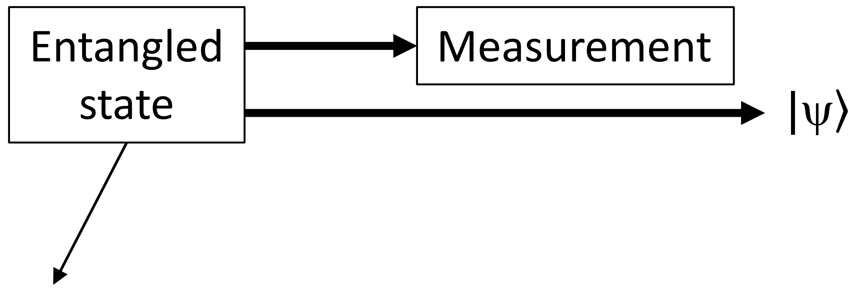
1. Projective measurements

2. Kerr-type non-linearities

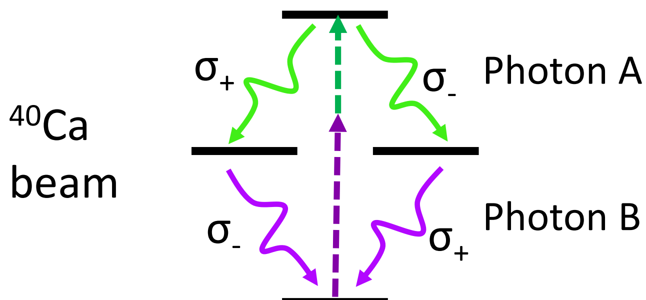
3. Single emitters

4. Rydberg atoms

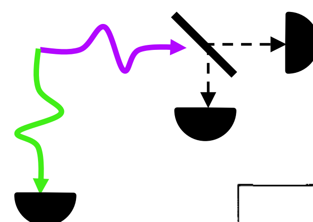
Projective measurements



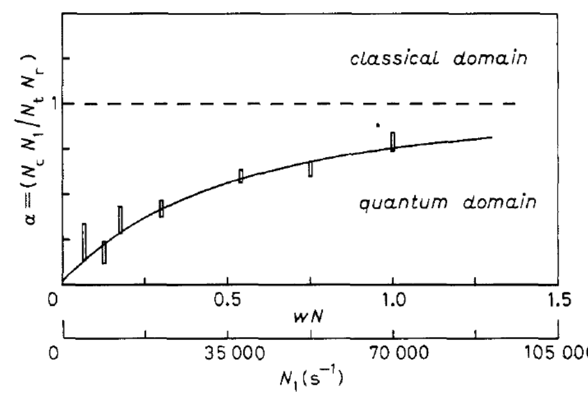
Freedman & Clauser, PRL **28**, 938 (1972),
Aspect, Grangier & Roger, PRL **47**, 460 (1981):



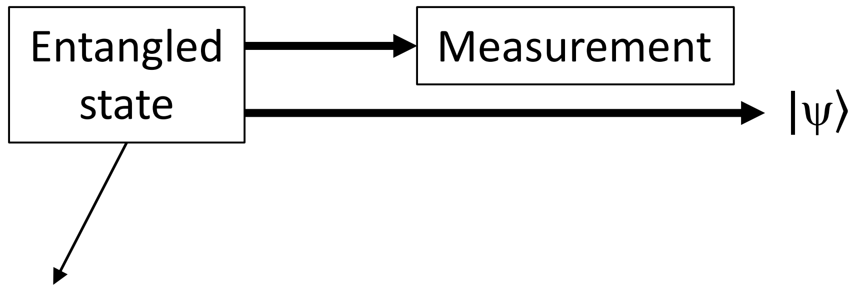
Grangier, Roger & Aspect,
EPL **1** 173 (1986):



$$g^{(2)} = \frac{\langle \hat{n}_1 \hat{n}_2 \rangle}{\langle \hat{n}_1 \rangle \langle \hat{n}_2 \rangle} < 1$$



Projective measurements



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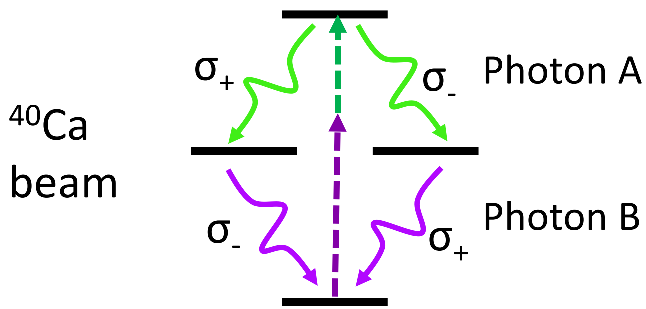

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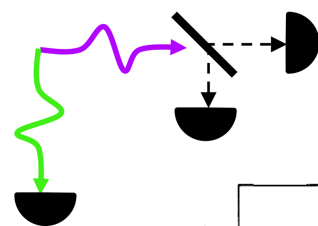

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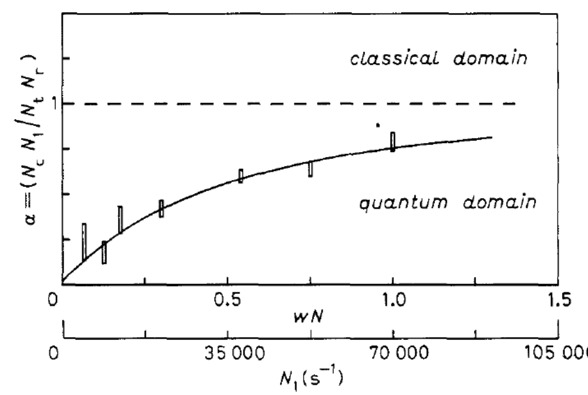
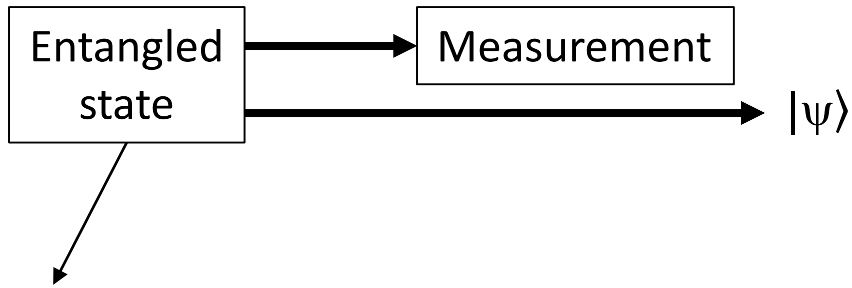


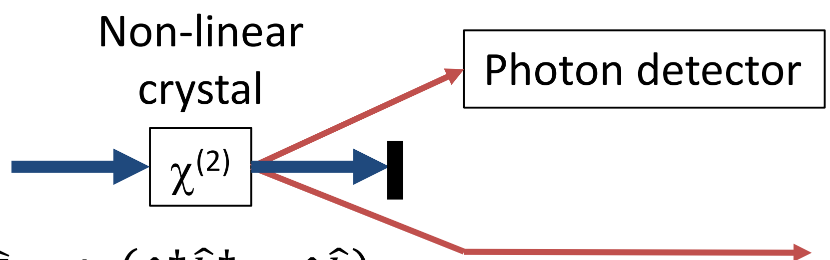
$\cancel{|1\rangle}$ $\rho=(1-\varepsilon)|0\rangle\langle 0|+\varepsilon|1\rangle\langle 1|$

$\langle 1|\rho|1\rangle \ll 1$

Spontaneous parametric downconversion

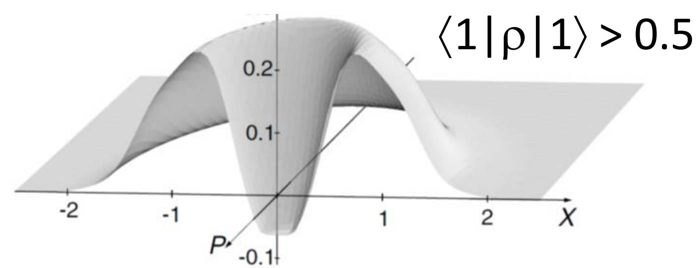


Hong & Mandel, PRL **56**, 58 (1986):



$$\hat{H} = i\chi\left(\hat{a}^{\dagger}\hat{b}^{\dagger} - \hat{a}\hat{b}\right)$$

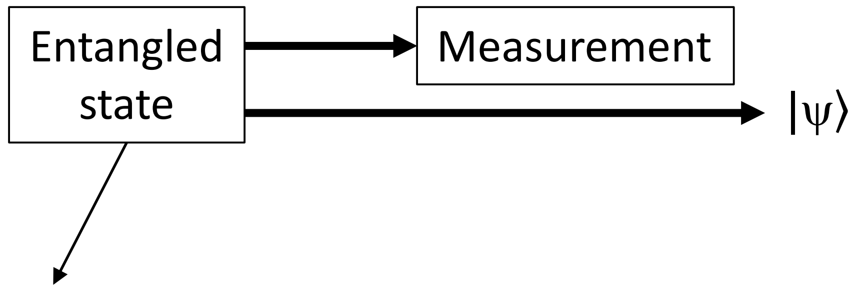
Lvovsky & al, PRL **87,** 050402 (2001):



Experimental review:
Lvovsky & al, arXiv:2006.16985

Theoretical review:
Walschaers, PRX Quantum **2,** 030204 (2021)

Spontaneous parametric downconversion



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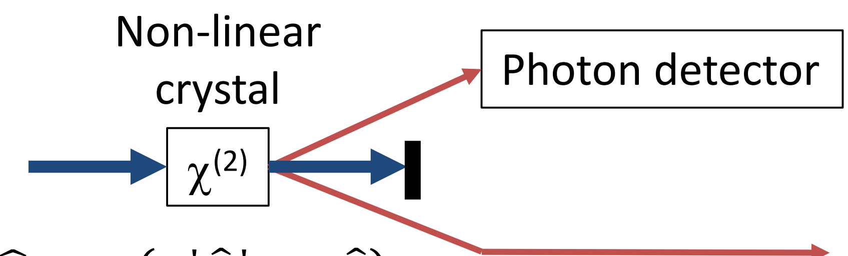


$$\hat{H} = i\chi\left(\hat{a}^\dagger \hat{b}^\dagger - \hat{a}\hat{b}\right)$$

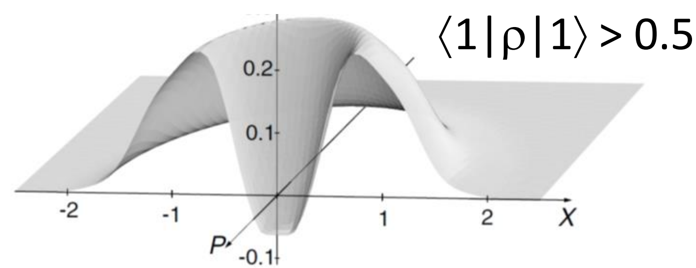
Exercise:

$$\Rightarrow |\psi\rangle = \sqrt{1-\lambda^2}\sum_{n=0}^{\infty} \lambda^n |n,n\rangle$$

$$\Rightarrow p_1 = (1-\lambda^2)\,\lambda^2 \leq 1/4$$

Bad, but not hopeless…

Lvovsky & al, PRL **87,** 050402 (2001):



Experimental review:
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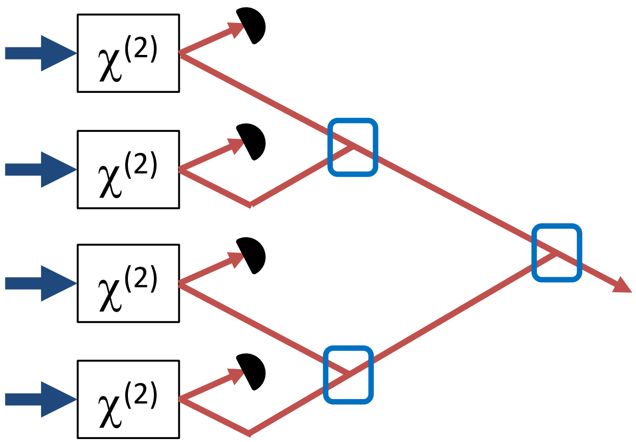
Spatial multiplexing

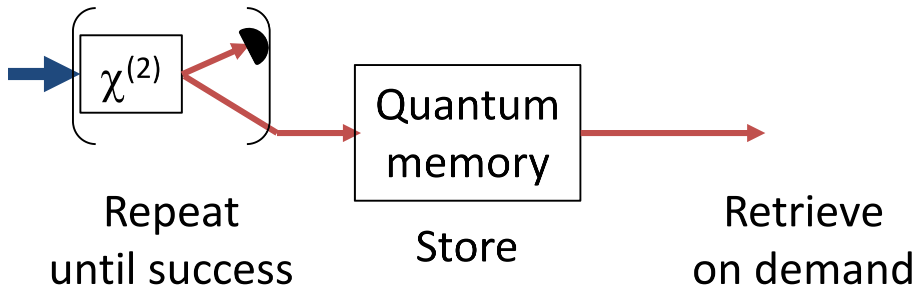


2^N sources

N layers
of active switches

	Free space	Integrated
Low losses		
Low noise		
Fast		

Temporal multiplexing



- High storage efficiency
- Low noise
- High retrieval efficiency
- Storage time >> 1/(success rate)

Temporal multiplexing

Duan, Lukin, Cirac & Zoller, Nature **414**, 413 (2001)

Read-only

Quantum memory

Project into singly-excited state

Retrieve on demand

- ~~High storage efficiency~~
- Low noise
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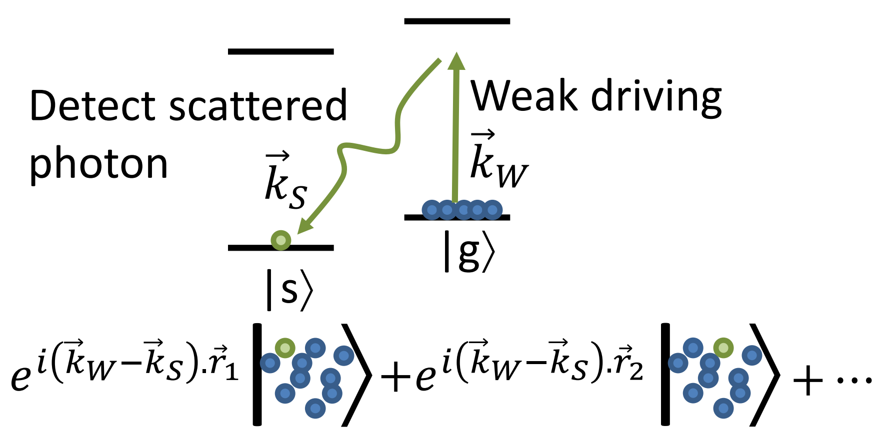

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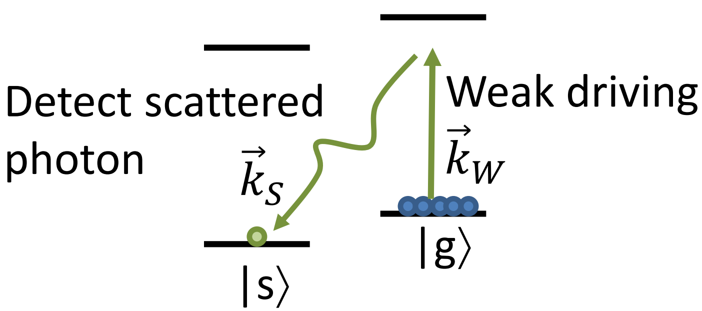
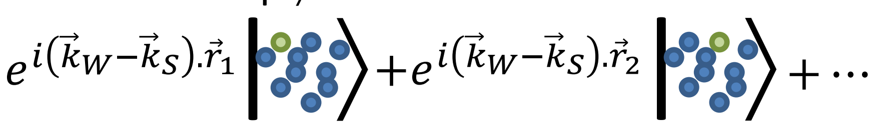
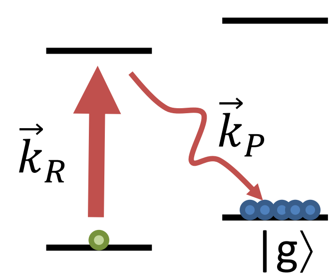






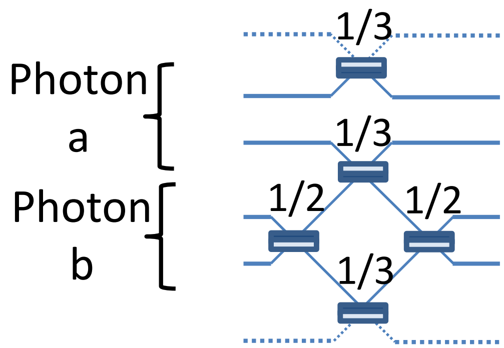
~ Quantum phased array antenna:

$$\vec{k}_P + \vec{k}_R = \vec{k}_S + \vec{k}_W$$

Linear optics quantum computation

Knill, Laflamme & Milburn, Nature 409, 46 (2001)

C-Not gate: O'Brien & al, Nature 426, 264 (2003)

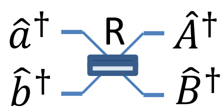


Linear optics quantum computation

Knill, Laflamme & Milburn, Nature 409, 46 (2001)

C-Not gate: O'Brien & al, Nature 426, 264 (2003)

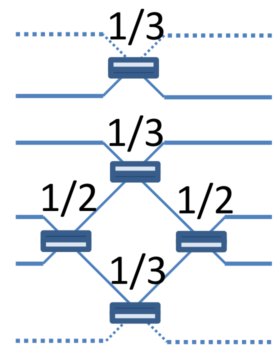
<u>Beamsplitter:</u>



$$\hat{a}^\dagger = \sqrt{R}\hat{A}^\dagger + \sqrt{1-R}\hat{B}^\dagger$$
$$\hat{b}^\dagger = \sqrt{1-R}\hat{A}^\dagger - \sqrt{R}\hat{B}^\dagger$$

Photon a

Photon b



<u>Postselection:</u>

one photon here

and

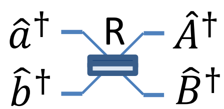
one photon here

Linear optics quantum computation

Knill, Laflamme & Milburn, Nature 409, 46 (2001)

C-Not gate: O'Brien & al, Nature 426, 264 (2003)

Beamsplitter:



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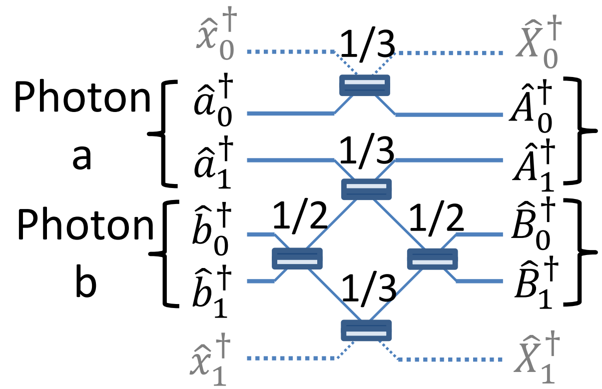


Postselection:

one photon here

and

one photon here

Exercise:

$$\hat{a}_0^\dagger \hat{b}_0^\dagger |0\rangle = \frac{1}{3}(\hat{A}_0^\dagger + \sqrt{2}X_0^\dagger)(\hat{A}_1^\dagger + \hat{B}_0^\dagger + \hat{X}_1^\dagger)|0\rangle \rightarrow \frac{1}{3}A_0^\dagger \hat{B}_0^\dagger |0\rangle$$

$$\hat{a}_0^\dagger \hat{b}_1^\dagger |0\rangle = \frac{1}{3}(\hat{A}_0^\dagger + \sqrt{2}\hat{X}_0^\dagger)(\hat{A}_1^\dagger + \hat{B}_1^\dagger - \hat{X}_1^\dagger)|0\rangle \rightarrow \frac{1}{3}\hat{A}_0^\dagger \hat{B}_1^\dagger |0\rangle$$

$$\hat{a}_1^\dagger \hat{b}_0^\dagger |0\rangle = \frac{1}{3}(-\hat{A}_1^\dagger + \hat{B}_0^\dagger + \hat{B}_1^\dagger)(\hat{A}_1^\dagger + \hat{B}_0^\dagger + \hat{X}_1^\dagger)|0\rangle \rightarrow \frac{1}{3}\hat{A}_1^\dagger \hat{B}_1^\dagger |0\rangle$$

$$\hat{a}_1^\dagger \hat{b}_1^\dagger |0\rangle = \frac{1}{3}(-\hat{A}_1^\dagger + \hat{B}_0^\dagger + \hat{B}_1^\dagger)(\hat{A}_1^\dagger + \hat{B}_1^\dagger - \hat{X}_1^\dagger)|0\rangle \rightarrow \frac{1}{3}\hat{A}_1^\dagger \hat{B}_0^\dagger |0\rangle$$

- Success probability = 1/9
- Need identical photons

Non-linear quantum optics

1. Projective measurements

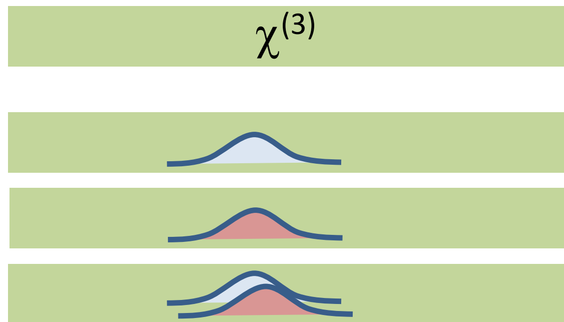
2. $\boxed{\text{Kerr-type non-linearities}}$

3. Single emitters

4. Rydberg atoms

Kerr-type non-linearities

Non-linear waveguide:
One photon changes the optical index seen by the other



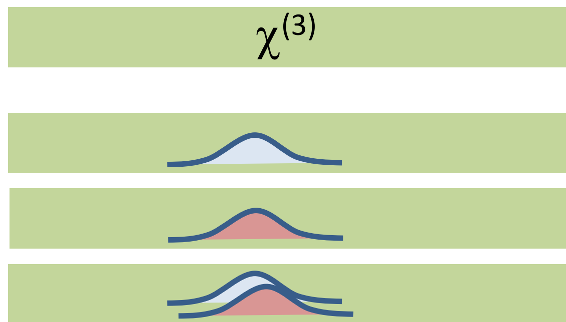
$|\psi_A\rangle|\psi_B\rangle$	$\exp(i\pi\hat{n}_A\hat{n}_B)|\psi_A\rangle|\psi_B\rangle$
$|1_A\rangle|0_B\rangle$	$|1_A\rangle|0_B\rangle$
$|0_A\rangle|1_B\rangle$	$|0_A\rangle|1_B\rangle$
$|1_A\rangle|1_B\rangle$	$-|1_A\rangle|1_B\rangle$

Popular approach: atomic gas in a hollow-core fiber
(very non-linear, strong transverse confinement, long distance)

Bajcsy & al, PRL **102**, 203902 (2009)

Kerr-type non-linearities

Non-linear waveguide:
One photon changes the optical index seen by the other



$|\psi_A\rangle|\psi_B\rangle \quad \longrightarrow \quad \cancel{\exp(i\pi\hat{n}_A\hat{n}_B)}|\psi_A\rangle|\psi_B\rangle$

$|1_A\rangle|0_B\rangle \quad \longrightarrow \quad |1_A\rangle|0_B\rangle$

$|0_A\rangle|1_B\rangle \quad \longrightarrow \quad |0_A\rangle|1_B\rangle$

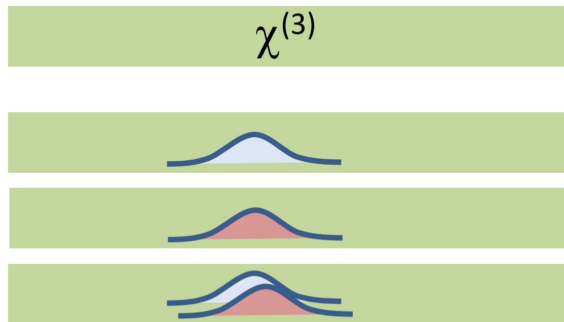
$|1_A\rangle|1_B\rangle \quad \longrightarrow \quad \cancel{-}|1_A\rangle|1_B\rangle$

J. Shapiro,
PRA **73**, 062305 (2006)

Kerr-type non-linearities

Non-linear waveguide:
One photon changes the optical index seen by the other

$|\psi_A\rangle|\psi_B\rangle$ → $\cancel{\exp(i\pi\hat{n}_A\hat{n}_B)}|\psi_A\rangle|\psi_B\rangle$



$|1_A\rangle|0_B\rangle$ → $|1_A\rangle|0_B\rangle$

$|0_A\rangle|1_B\rangle$ → $|0_A\rangle|1_B\rangle$

$|1_A\rangle|1_B\rangle$ → $\cancel{-}|1_A\rangle|1_B\rangle$

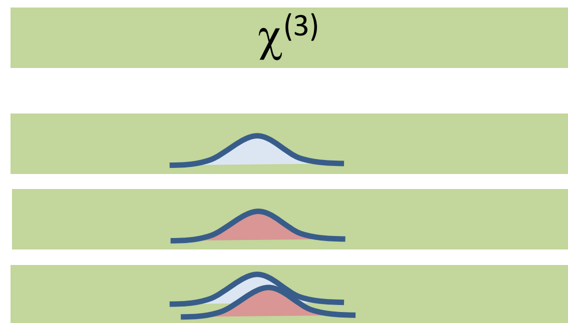
↓

Must be in the same place at the same time

J. Shapiro,
PRA **73**, 062305 (2006)

Kerr-type non-linearities

Non-linear waveguide:
One photon changes the optical index seen by the other



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Must be in the same place at the same time

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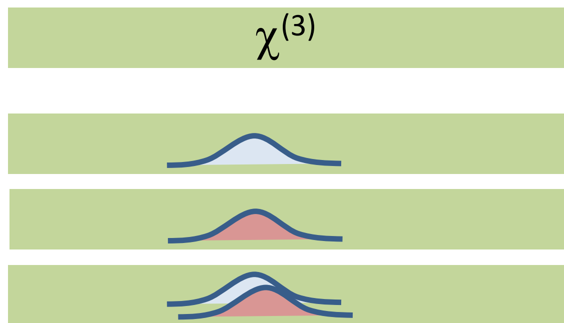
Photon duration < System's response time

J. Shapiro,
PRA **73**, 062305 (2006)

Kerr-type non-linearities

Non-linear waveguide:
One photon changes the optical index seen by the other



$\|\psi_A\rangle\|\psi_B\rangle$	$\cancel{\exp(i\pi\hat{n}_A\hat{n}_B)}\|\psi_A\rangle\|\psi_B\rangle$
$\|1_A\rangle\|0_B\rangle$	$\|1_A\rangle\|0_B\rangle$
$\|0_A\rangle\|1_B\rangle$	$\|0_A\rangle\|1_B\rangle$
$\|1_A\rangle\|1_B\rangle$	$\cancel{-}\|1_A\rangle\|1_B\rangle$

↓

Must be in the same place at the same time

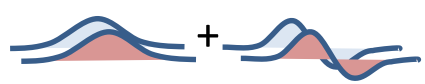
↓

Photon duration < System's response time

↓

Photon spectral width > System's bandwidth

↓



+...

J. Shapiro,
PRA **73**, 062305 (2006)

Kerr-type non-linearities

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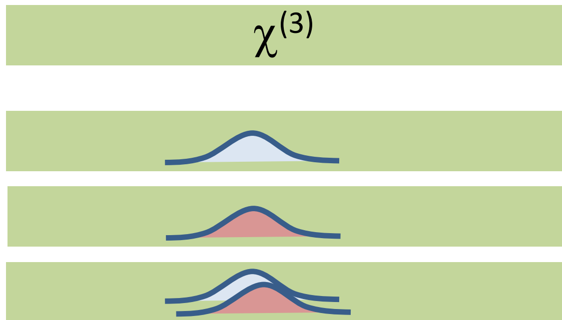

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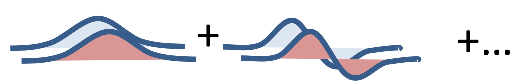


J. Shapiro,
PRA **73**, 062305 (2006)

« What is proved by impossibility proofs is lack of imagination » (J.S. Bell)

Non-linear quantum optics

1. Projective measurements

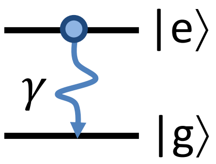
2. Kerr-type non-linearities

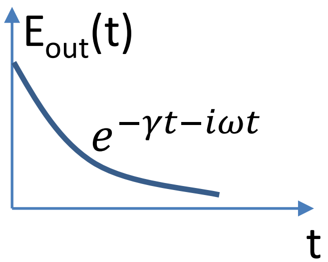
3. Single emitters

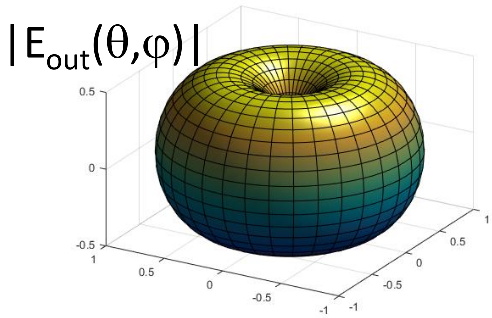
4. Rydberg atoms

Single Atoms

Simplest source of pure non-Gaussian states:

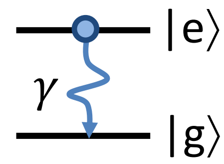


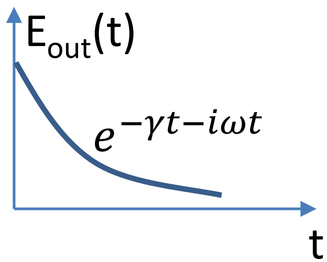


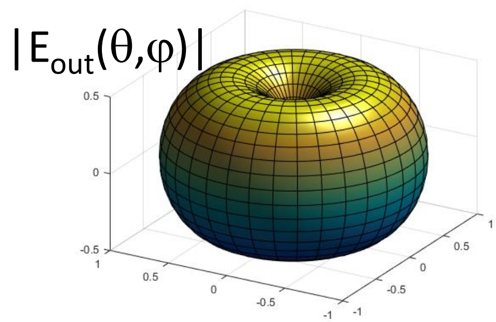


Single Atoms

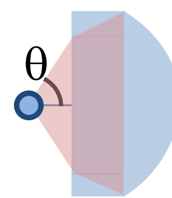
Simplest source of pure non-Gaussian states:







Photon collection efficiency η:



NA=sin(θ)=0.92
$\Rightarrow$ Collection 30%,
Fiber coupling 30%

Bakr &al, Nature **462**, 74 (2009)
Robens &al, Opt. Lett. **42**, 1043 (2017)
Carter &al, Rev. Sci. Instr. **95**, 033201 (2014)

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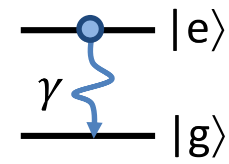

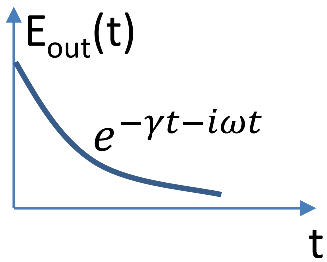

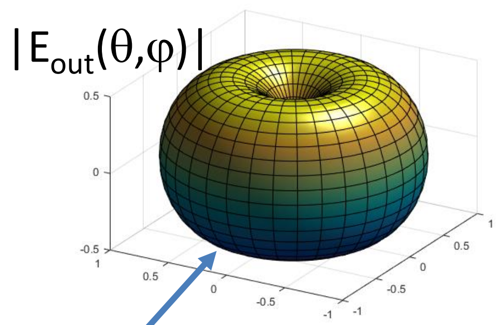

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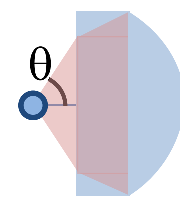

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To change this shape,
change the density of states
around the atom

Bakr &al, Nature **462**, 74 (2009)
Robens &al, Opt. Lett. **42**, 1043 (2017)
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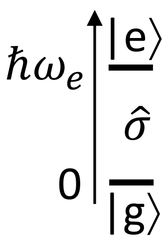
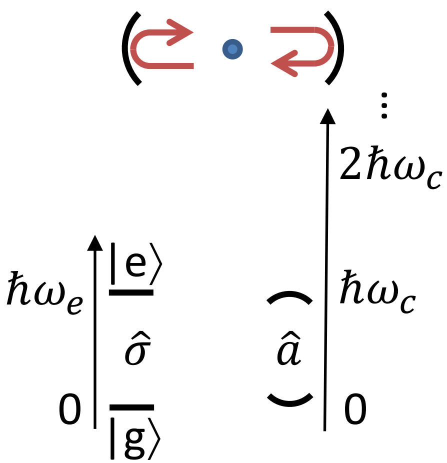
Single Atoms

$$\hat{H} = \underbrace{\hbar\omega_e\hat{\sigma}^\dagger\hat{\sigma}}_{\boxed{\hat{\sigma} = |g\rangle\langle e|}}$$

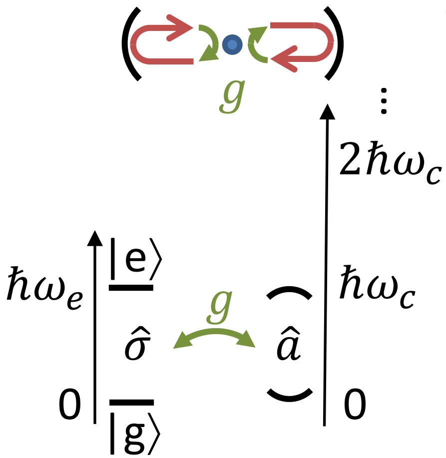


Single Atoms



$$\hat{H} = \underbrace{\hbar\omega_e \hat{\sigma}^\dagger \hat{\sigma}}_{\boxed{\hat{\sigma} = |g\rangle\langle e|}} + \underbrace{\hbar\omega_c \hat{a}^\dagger \hat{a}}_{\boxed{[\hat{a}, \hat{a}^\dagger] = 1}}$$

Single Atoms



Jaynes-Cummings

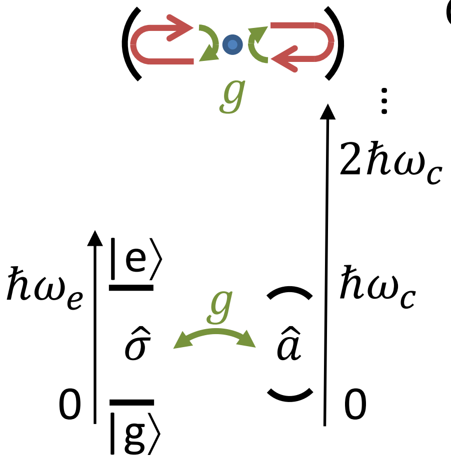
$$\hat{H} = \underbrace{\hbar\omega_e\hat{\sigma}^\dagger\hat{\sigma}}_{\boxed{\hat{\sigma} = |g\rangle\langle e|}} + \underbrace{\hbar\omega_c\hat{a}^\dagger\hat{a}}_{\boxed{[\hat{a},\hat{a}^\dagger] = 1}} + \underbrace{\hbar g(\hat{a}^\dagger\hat{\sigma} + \hat{\sigma}^\dagger\hat{a})}$$

$$-\vec{d}\,(\hat{\sigma}+\hat{\sigma}^\dagger).\vec{E}(\vec{r})(\hat{a}+\hat{a}^\dagger)$$

$$\approx \hbar\left(-\frac{\vec{d}.\vec{E}(\vec{r}_0)}{\hbar}\right)(\hat{a}^\dagger\hat{\sigma} + \hat{\sigma}^\dagger\hat{a})$$

$$(\omega_e, \omega_c \gg g)$$

Single Atoms

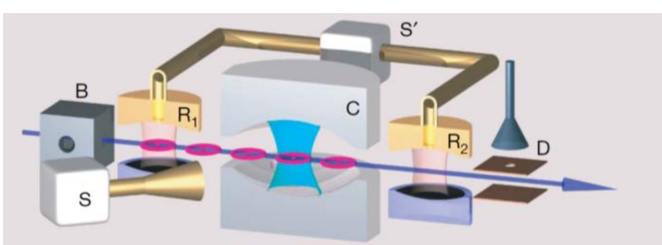


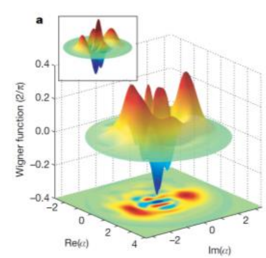
Jaynes-Cummings

$$\hat H = \underbrace{\hbar\omega_e\hat\sigma^\dagger\hat\sigma}_{\boxed{\hat\sigma = |g\rangle\langle e|}} + \underbrace{\hbar\omega_c\hat a^\dagger\hat a}_{\boxed{[\hat a, \hat a^\dagger] = 1}} + \underbrace{\hbar g(\hat a^\dagger\hat\sigma + \hat\sigma^\dagger\hat a)}_{\boxed{\begin{array}{r} -\vec d\,(\hat\sigma + \hat\sigma^\dagger).\vec E(\vec r)(\hat a + \hat a^\dagger) \\ \approx \hbar\left(-\dfrac{\vec d.\vec E(\vec r_0)}{\hbar}\right)(\hat a^\dagger\hat\sigma + \hat\sigma^\dagger\hat a) \\ (\omega_e, \omega_c \gg g) \end{array}}}$$

Trapped microwave photons:

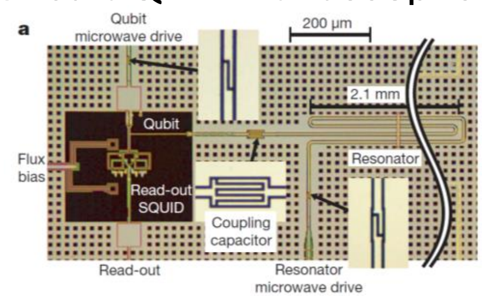
Cavity QED with Rydberg atoms

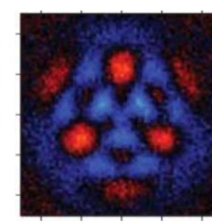




Deléglise &al, Nature **455**, 510 (2008)

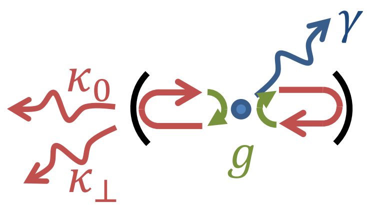
Circuit QED with Josephson junctions

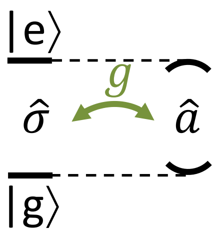




Hofheinz &al, Nature **459**, 546 (2009)

Single Atoms





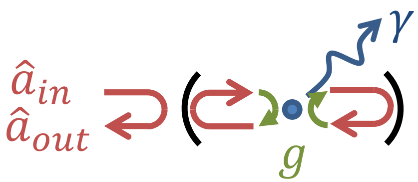
Frame rotating at $\omega=\omega_e=\omega_c$, $\kappa=\kappa_0+\kappa_\perp$:

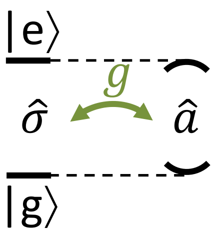
$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right] + \mathcal{L}_{\sqrt{2\kappa}\hat{a}}(\hat{\rho}) + \mathcal{L}_{\sqrt{2\gamma}\hat{\sigma}_{ge}}(\hat{\rho})$$

$$\frac{\hat{H}}{\hbar} = g\left(\hat{a}^\dagger\hat{\sigma}_{ge} + \hat{\sigma}_{ge}^\dagger\hat{a}\right)$$

$$\mathcal{L}_{\hat{O}}(\hat{\rho}) = \hat{O}\hat{\rho}\hat{O}^\dagger - \frac{1}{2}\left\{\hat{O}^\dagger\hat{O},\hat{\rho}\right\}$$

Single Atoms





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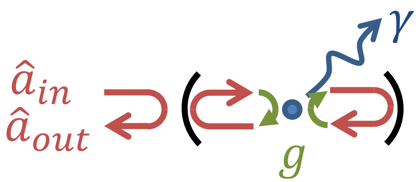
Single Atoms



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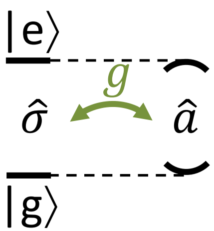

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⇒ Photon collection efficiency:

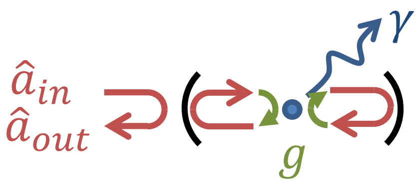
Exercise:

$$\hat{\rho}(0) = |e,0\rangle\langle e,0|,\ \left\langle\hat{a}_{in}^\dagger\hat{a}_{in}\right\rangle = 0$$

$$\boxed{\Rightarrow \eta = \int_0^\infty \left\langle\hat{a}_{out}^\dagger\hat{a}_{out}\right\rangle dt = \frac{\kappa_0}{\kappa+\gamma}\frac{2C}{1+2C}}$$

$$\boxed{\text{Cooperativity } C = \frac{g^2}{2\kappa\gamma}}$$

Single Atoms



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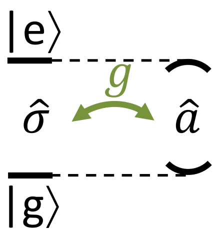

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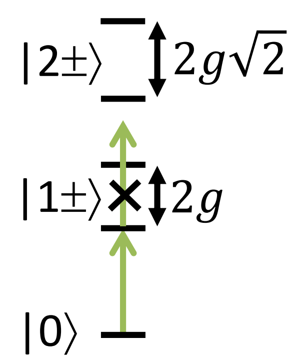
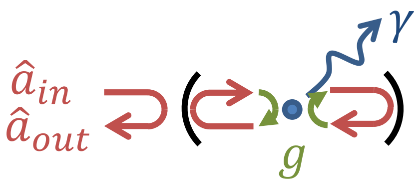
⇒ Two-photon gate?



Single Atoms



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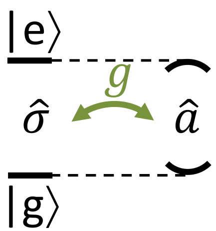

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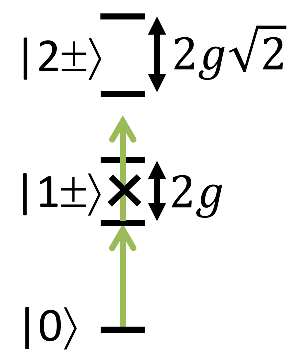
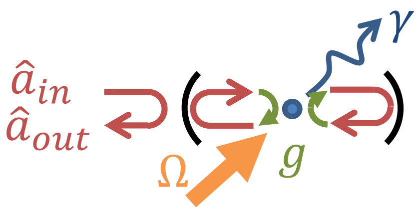
⇒ ~~Two-photon gate~~? Like $\chi^{(3)}$, need active control



Kiilerich & Mølmer, PRA **102**, 023717(2020)

Single Atoms



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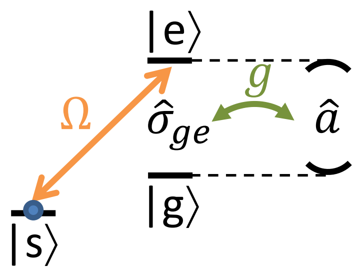

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$\Rightarrow$ Photon collection efficiency:

Exercise:

$$\hat{\rho}(0) = |s,0\rangle\langle s,0|,\ \langle\hat{a}_{in}^\dagger\hat{a}_{in}\rangle = 0, \boxed{\Omega \ll g}$$

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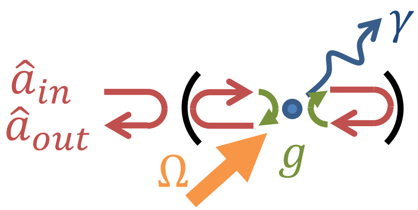
Gorshkov &al,
PRA **76**, 033804 (2007):
$\boxed{\kappa \gg g}$

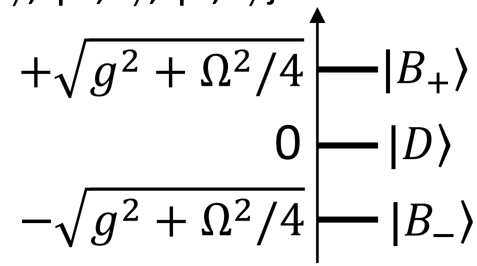
Stanojevic &al,
PRA **84**, 053830 (2011):
$\boxed{\dot{\Omega}/\Omega \ll g}$

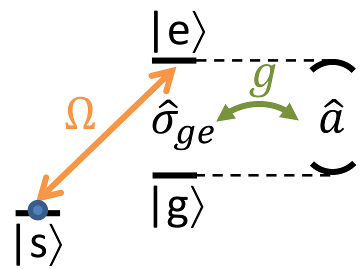
Single Atoms



Single-excitation subspace {|g,1⟩, |e,0⟩, |s,0⟩}:

$$\frac{\hat{H}}{\hbar} = \begin{pmatrix} 0 & g & 0 \\ g & 0 & \Omega/2 \\ 0 & \Omega/2 & 0 \end{pmatrix}$$





$$|D\rangle = \cos(\beta)|g,1\rangle - \sin(\beta)\,|s,0\rangle$$

$$\tan(\beta) = 2g/\Omega$$

Adiabatic rotation of |D⟩
→ $\hat{a}_{out}$(t) controlled by Ω(t)

⇒ Photon collection efficiency:

Exercise:

$$\hat{\rho}(0) = |s,0\rangle\langle s,0|,\ \langle \hat{a}_{in}^\dagger \hat{a}_{in}\rangle = 0,\ \boxed{\Omega \ll g}$$

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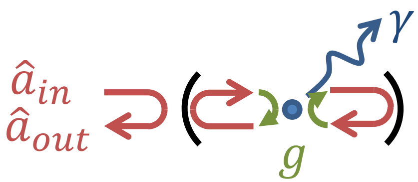
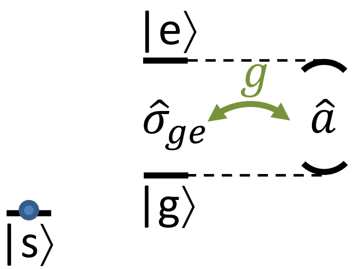




Assumptions:

$C = \frac{g^2}{2\kappa\gamma} \gg 1,$

$\kappa = \kappa_0,$

$\hat{a}_{in}$ weak & slow.

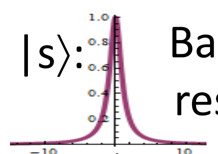
Exercise:

$$\langle \dot{\hat{a}} \rangle = -ig\langle \hat{\sigma}_{ge} \rangle - \kappa\langle \hat{a} \rangle - \sqrt{2\kappa}\langle \hat{a}_{in} \rangle \approx 0$$

$$\langle \dot{\hat{\sigma}}_{ge} \rangle \approx -ig\langle \hat{a} \rangle - \gamma\langle \hat{\sigma}_{ge} \rangle \approx 0$$

$$\langle \hat{a}_{out} \rangle = \langle \hat{a}_{in} \rangle + \sqrt{2\kappa}\langle \hat{a} \rangle$$

$|s\rangle$:

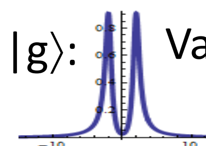


Bare cavity resonance

$\downarrow$

$$\frac{\langle \hat{a}_{out} \rangle}{\langle \hat{a}_{in} \rangle} = -1$$

$|g\rangle$:

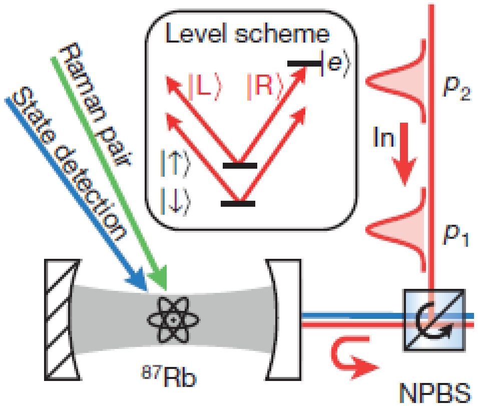


Vacuum Rabi splitting

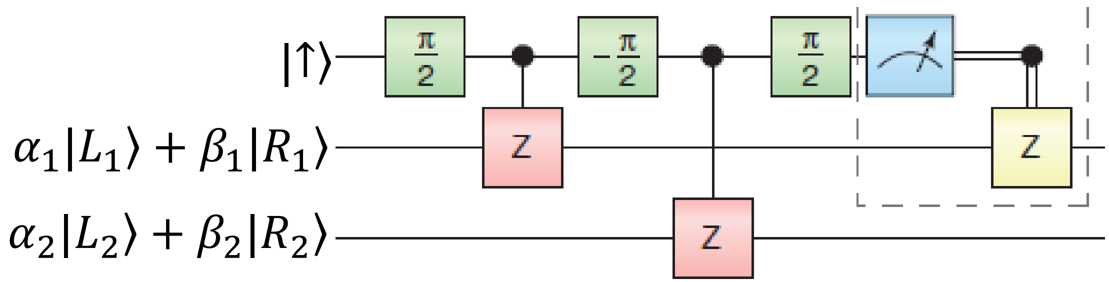
$\downarrow$

$$\frac{\langle \hat{a}_{out} \rangle}{\langle \hat{a}_{in} \rangle} = 1 - \frac{2}{1+2C} \approx 1$$

Single Atoms



⇒ Two-photon gate: Hacker & al, Nature **536**, 193 (2016)



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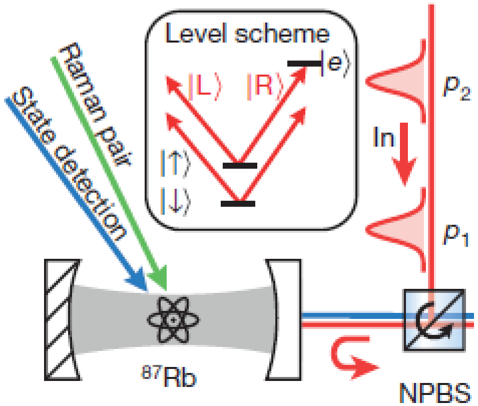
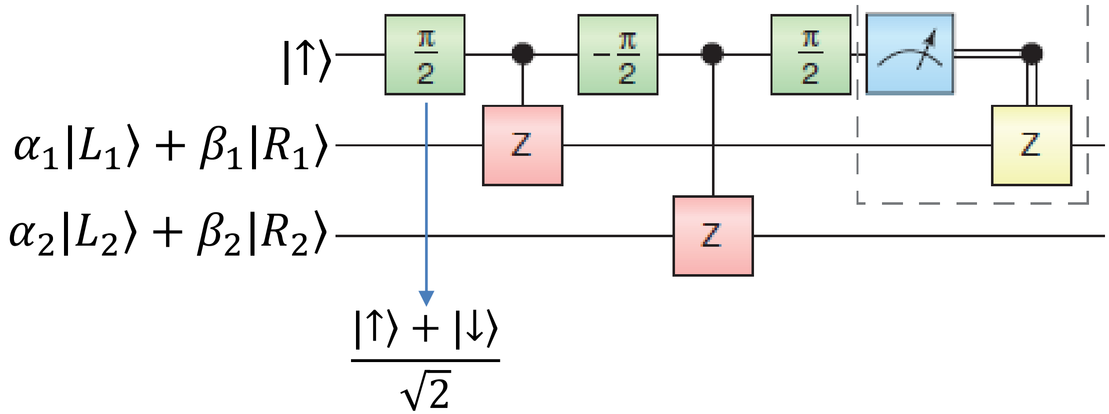
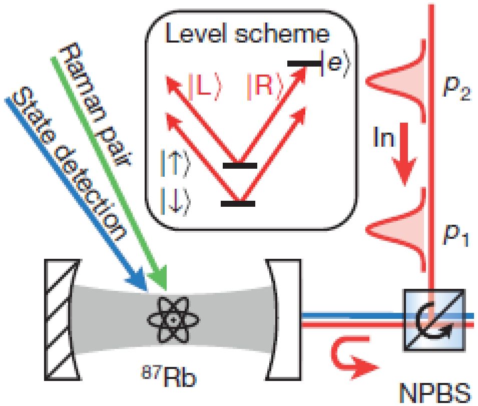


$\Rightarrow$ Two-photon gate: Hacker & al, Nature **536**, 193 (2016)



Single Atoms



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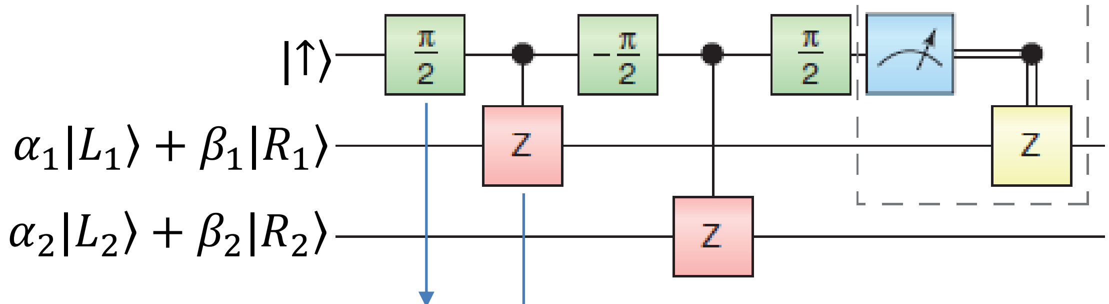
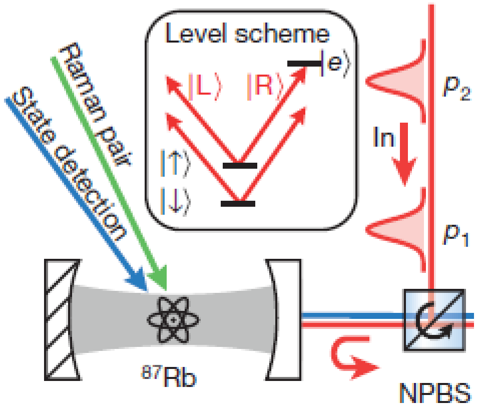


$$\frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}$$

$$\frac{\alpha_1}{\sqrt{2}}|\uparrow\rangle|L_1\rangle - \frac{\beta_1}{\sqrt{2}}|\uparrow\rangle|R_1\rangle + \frac{\alpha_1}{\sqrt{2}}|\downarrow\rangle|L_1\rangle + \frac{\beta_1}{\sqrt{2}}|\downarrow\rangle|R_1\rangle$$

Single Atoms

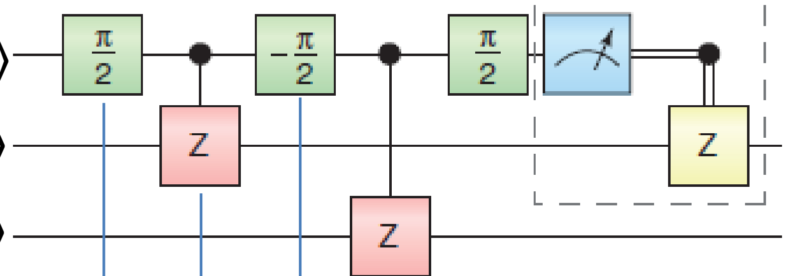


$\Rightarrow$ Two-photon gate: Hacker & al, Nature **536**, 193 (2016)

$|\uparrow\rangle$

$\alpha_1|L_1\rangle + \beta_1|R_1\rangle$

$\alpha_2|L_2\rangle + \beta_2|R_2\rangle$

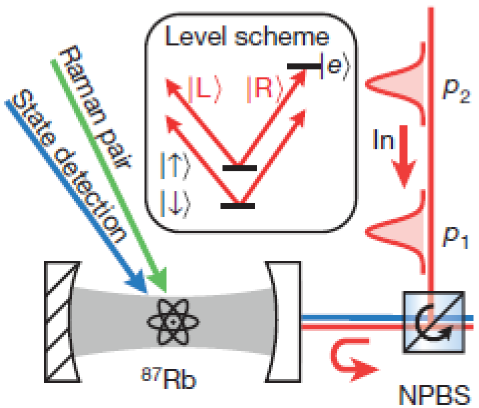


$$\frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}$$

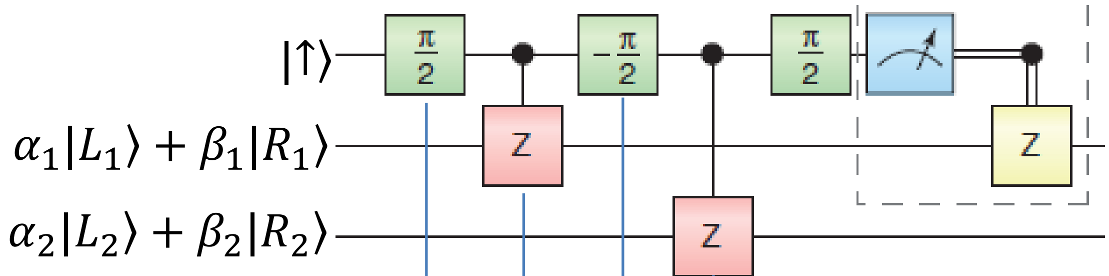
$$\frac{\alpha_1}{\sqrt{2}}|\uparrow\rangle|L_1\rangle - \frac{\beta_1}{\sqrt{2}}|\uparrow\rangle|R_1\rangle + \frac{\alpha_1}{\sqrt{2}}|\downarrow\rangle|L_1\rangle + \frac{\beta_1}{\sqrt{2}}|\downarrow\rangle|R_1\rangle$$

$$\alpha_1|\uparrow\rangle|L_1\rangle + \beta_1|\downarrow\rangle|R_1\rangle$$

Single Atoms



$\Rightarrow$ Two-photon gate: Hacker & al, Nature **536**, 193 (2016)



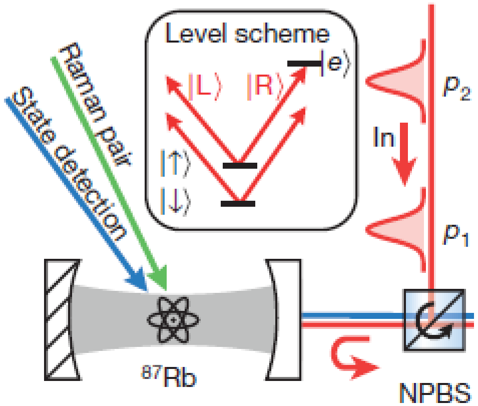
$$\frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}$$

$$\frac{\alpha_1}{\sqrt{2}}|\uparrow\rangle|L_1\rangle - \frac{\beta_1}{\sqrt{2}}|\uparrow\rangle|R_1\rangle + \frac{\alpha_1}{\sqrt{2}}|\downarrow\rangle|L_1\rangle + \frac{\beta_1}{\sqrt{2}}|\downarrow\rangle|R_1\rangle$$

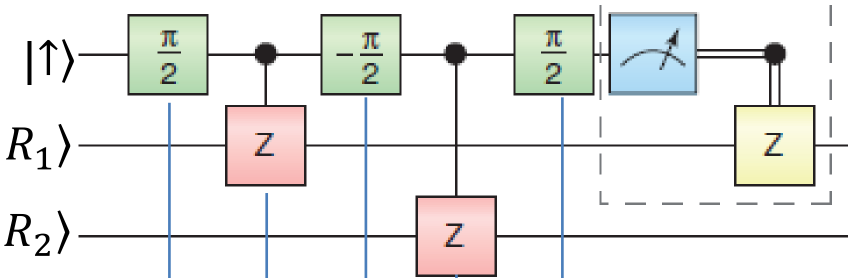
$$\alpha_1|\uparrow\rangle|L_1\rangle + \beta_1|\downarrow\rangle|R_1\rangle$$

$$\alpha_1\alpha_2|\uparrow\rangle|L_1\rangle|L_2\rangle - \alpha_1\beta_2|\uparrow\rangle|L_1\rangle|R_2\rangle + \beta_1\alpha_2|\downarrow\rangle|R_1\rangle|L_2\rangle + \beta_1\beta_2|\downarrow\rangle|R_1\rangle|R_2\rangle$$

Single Atoms



$\Rightarrow$ Two-photon gate: Hacker & al, Nature **536**, 193 (2016)



$$\frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}$$

$$\frac{\alpha_1}{\sqrt{2}}|\uparrow\rangle|L_1\rangle - \frac{\beta_1}{\sqrt{2}}|\uparrow\rangle|R_1\rangle + \frac{\alpha_1}{\sqrt{2}}|\downarrow\rangle|L_1\rangle + \frac{\beta_1}{\sqrt{2}}|\downarrow\rangle|R_1\rangle$$

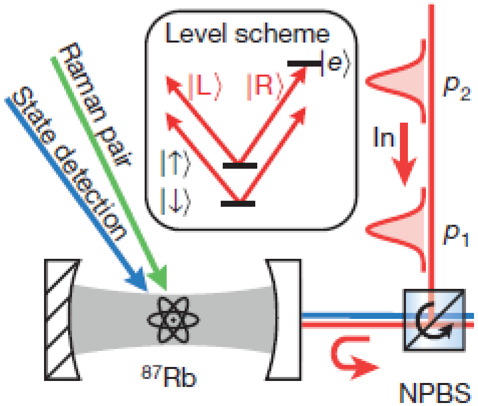
$$\alpha_1|\uparrow\rangle|L_1\rangle + \beta_1|\downarrow\rangle|R_1\rangle$$

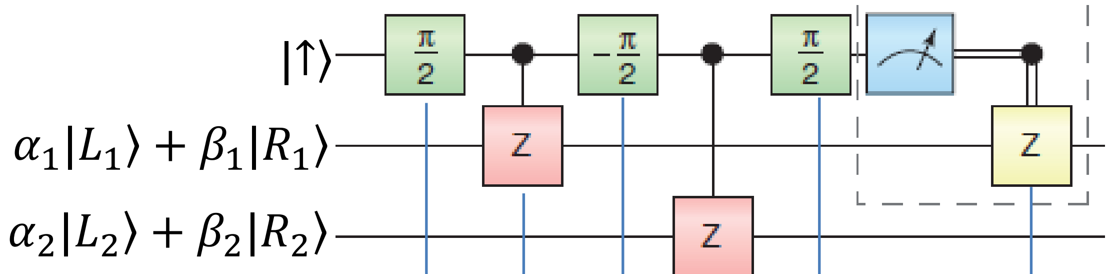
$$\alpha_1\alpha_2|\uparrow\rangle|L_1\rangle|L_2\rangle - \alpha_1\beta_2|\uparrow\rangle|L_1\rangle|R_2\rangle + \beta_1\alpha_2|\downarrow\rangle|R_1\rangle|L_2\rangle + \beta_1\beta_2|\downarrow\rangle|R_1\rangle|R_2\rangle$$

$$|\uparrow\rangle(\alpha_1\alpha_2|L_1\rangle|L_2\rangle - \alpha_1\beta_2|L_1\rangle|R_2\rangle - \beta_1\alpha_2|R_1\rangle|L_2\rangle - \beta_1\beta_2|R_1\rangle|R_2\rangle)/\sqrt{2}$$
$$+|\downarrow\rangle(\alpha_1\alpha_2|L_1\rangle|L_2\rangle - \alpha_1\beta_2|L_1\rangle|R_2\rangle + \beta_1\alpha_2|R_1\rangle|L_2\rangle + \beta_1\beta_2|R_1\rangle|R_2\rangle)/\sqrt{2}$$

Single Atoms

$\Rightarrow$ Two-photon gate: Hacker & al, Nature **536**, 193 (2016)





$$\frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}$$

$$\frac{\alpha_1}{\sqrt{2}}|\uparrow\rangle|L_1\rangle - \frac{\beta_1}{\sqrt{2}}|\uparrow\rangle|R_1\rangle + \frac{\alpha_1}{\sqrt{2}}|\downarrow\rangle|L_1\rangle + \frac{\beta_1}{\sqrt{2}}|\downarrow\rangle|R_1\rangle$$

If $|\uparrow\rangle$, $|R_1\rangle \rightarrow -|R_1\rangle$

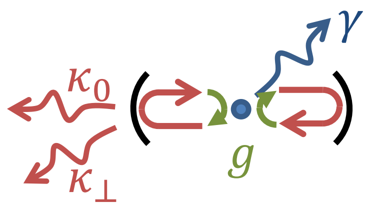
$$\alpha_1|\uparrow\rangle|L_1\rangle + \beta_1|\downarrow\rangle|R_1\rangle$$

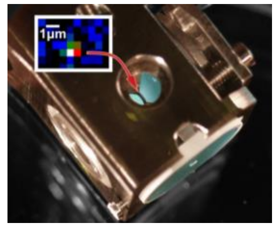
$$\alpha_1\alpha_2|\uparrow\rangle|L_1\rangle|L_2\rangle - \alpha_1\beta_2|\uparrow\rangle|L_1\rangle|R_2\rangle + \beta_1\alpha_2|\downarrow\rangle|R_1\rangle|L_2\rangle + \beta_1\beta_2|\downarrow\rangle|R_1\rangle|R_2\rangle$$

$$|\uparrow\rangle(\alpha_1\alpha_2|L_1\rangle|L_2\rangle - \alpha_1\beta_2|L_1\rangle|R_2\rangle - \beta_1\alpha_2|R_1\rangle|L_2\rangle - \beta_1\beta_2|R_1\rangle|R_2\rangle)/\sqrt{2}$$
$$+|\downarrow\rangle(\alpha_1\alpha_2|L_1\rangle|L_2\rangle - \alpha_1\beta_2|L_1\rangle|R_2\rangle + \beta_1\alpha_2|R_1\rangle|L_2\rangle + \beta_1\beta_2|R_1\rangle|R_2\rangle)/\sqrt{2}$$

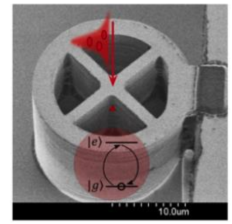
$$\alpha_1\alpha_2|L_1\rangle|L_2\rangle - \alpha_1\beta_2|L_1\rangle|R_2\rangle + \beta_1\alpha_2|R_1\rangle|L_2\rangle + \beta_1\beta_2|R_1\rangle|R_2\rangle$$

Single « atoms » in cavities

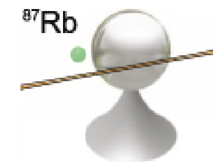




Rempe@MPQ



Senellart@C2N



Dayan@Weizmann

Key figures of merit:

- **Cavity extraction efficiency**

$$\eta_{cav} = \frac{\kappa_0}{\kappa_0 + \kappa_\perp} = \frac{Transmission}{Transmission + Losses}$$

Challenge: mirror substrates & coatings

- **Cooperativity**

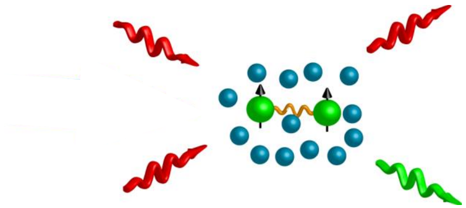
$$C_0 = \frac{g^2}{2(\kappa_0 + \kappa_\perp)\gamma} = \frac{1}{Transmission + Losses} * \frac{\sigma_{Atom} \sim \lambda^2}{\sigma_{Photon} \sim w^2}$$

Challenge: controlling atom-cavity coupling

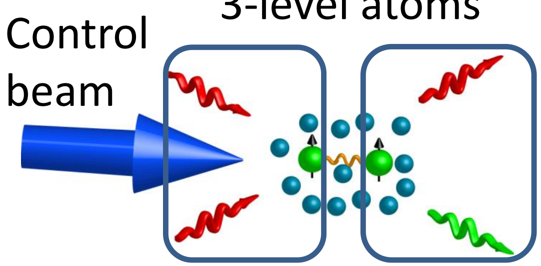
Non-linear quantum optics

1. Projective measurements
2. Kerr-type non-linearities
3. Single emitters
4. Rydberg atoms

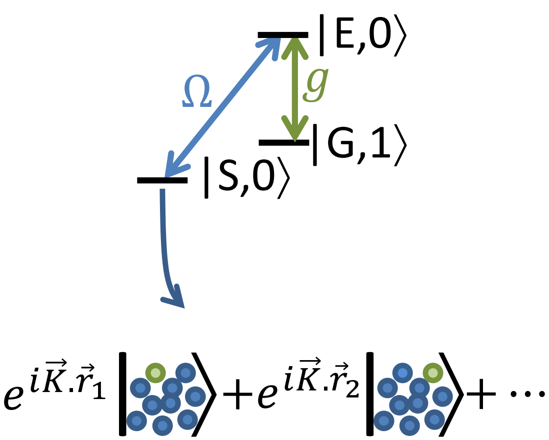
Rydberg Atoms



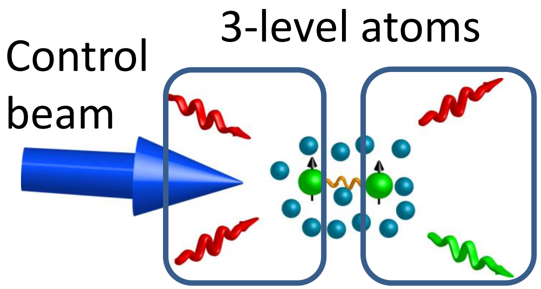
Rydberg Atoms



~ Quantum Memory



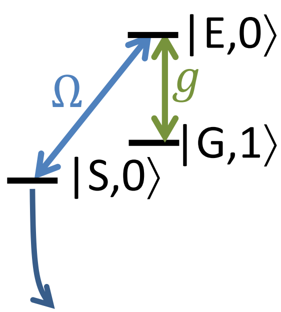
Rydberg Atoms

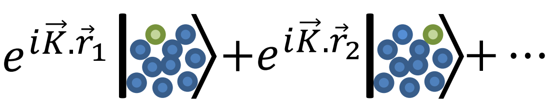


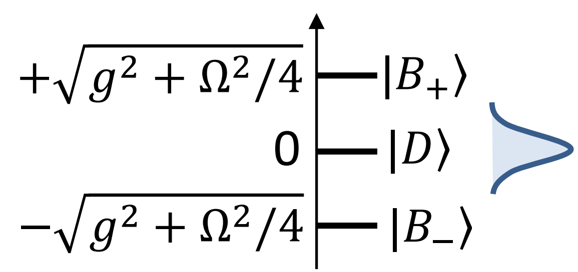
~ Quantum Memory

$g_0\sqrt{N_{atoms}}$

$$\frac{\hat{H}}{\hbar} = \begin{pmatrix} 0 & g & 0 \\ g & 0 & \Omega/2 \\ 0 & \Omega/2 & 0 \end{pmatrix}$$







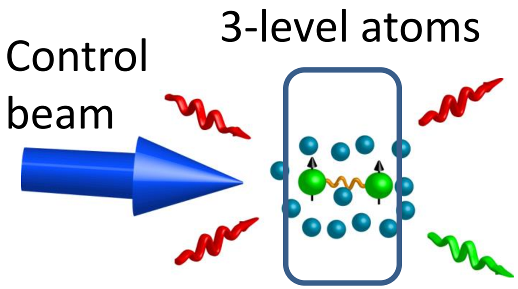
Dark polariton

$$|D\rangle = \cos(\beta)\,|G,1\rangle - \sin(\beta)\,|S,0\rangle$$

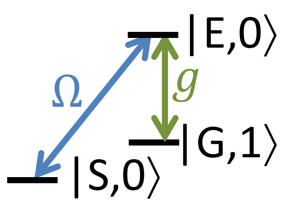
$$\tan(\beta) = \frac{2g}{\Omega}$$

Electromagnetically-Induced Transparency (EIT)

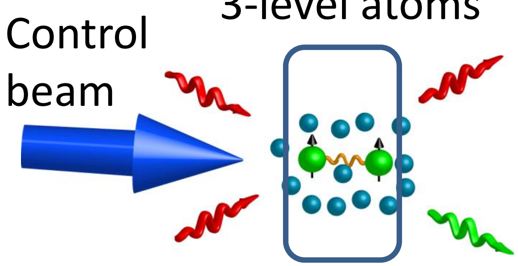
Rydberg Atoms

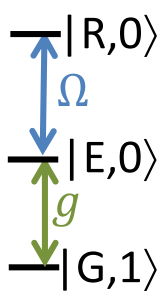


Ground-state interactions: ~100 Hz @ ~0.1 μm



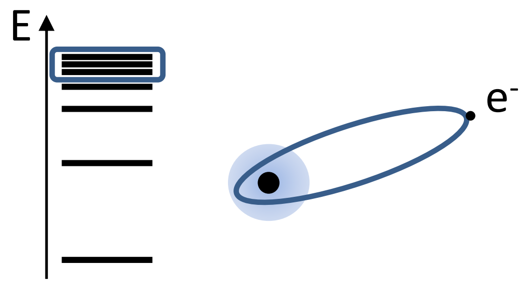
Rydberg Atoms





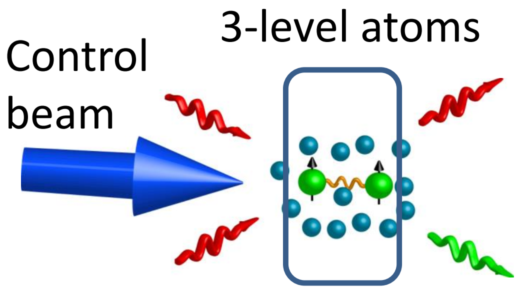
Ground-state interactions: ~100 Hz @ ~0.1 μm

Rydberg atom: ~Hydrogen, n*=n-δ, δ<<n (quantum defect)



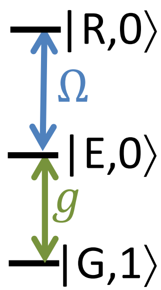
Quantity	Scaling	n~100
Energy	$-Ry/{n^*}^2$	-100 GHz
Level splitting Δ	$Ry/{n^*}^3$	1 GHz
Ryd-Ryd dipole d	$e\,a_0\,{n^*}^2$	10^4*water
Lifetime (low l)	$\propto {n^*}^3$(0K)	100 μs

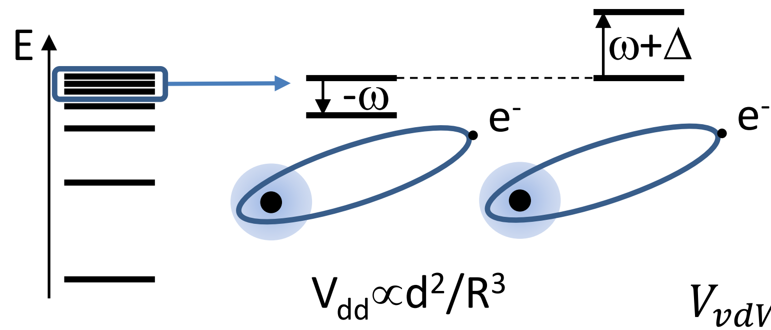
Rydberg Atoms



Ground-state interactions: ~100 Hz @ ~0.1 μm

Rydberg atom: ~Hydrogen, $n^*=n-\delta$, $\delta << n$ (quantum defect)





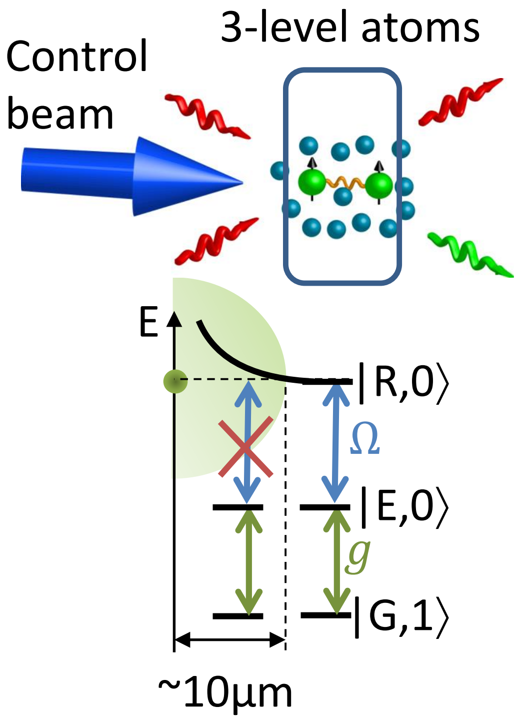
$$H = \begin{pmatrix} 0 & V_{dd} \\ V_{dd} & \hbar\Delta \end{pmatrix}$$

$\hbar\Delta \gg V_{dd}$:

$$V_{vdW} \simeq -\frac{{V_{dd}}^2}{\hbar\Delta} \propto \frac{n^{*11}}{R^6}$$

Quantity	Scaling	n~100
Energy	$-Ry\ /{n^*}^2$	-100 GHz
Level splitting Δ	$Ry\ /{n^*}^3$	1 GHz
Ryd-Ryd dipole d	$e\ a_0\ {n^*}^2$	10^4*water
Lifetime (low l)	$\propto {n^*}^3$(0K)	100 μs
Interactions V_{vdW}	$\propto {n^*}^{11}/R^6$	100 **MHz** @ 10 **μm**

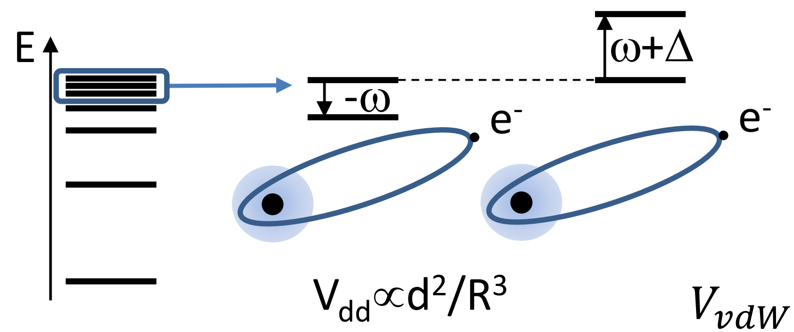
Rydberg Atoms



V_{vdW}> linewidth:
Rydberg blockade

Ground-state interactions: ~100 Hz @ ~0.1 µm

Rydberg atom: ~Hydrogen, n*=n-δ, δ<<n (quantum defect)



$$H = \begin{pmatrix} 0 & V_{dd} \\ V_{dd} & \hbar\Delta \end{pmatrix}$$

$\hbar\Delta \gg V_{dd}$:

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Quantity	Scaling	n~100
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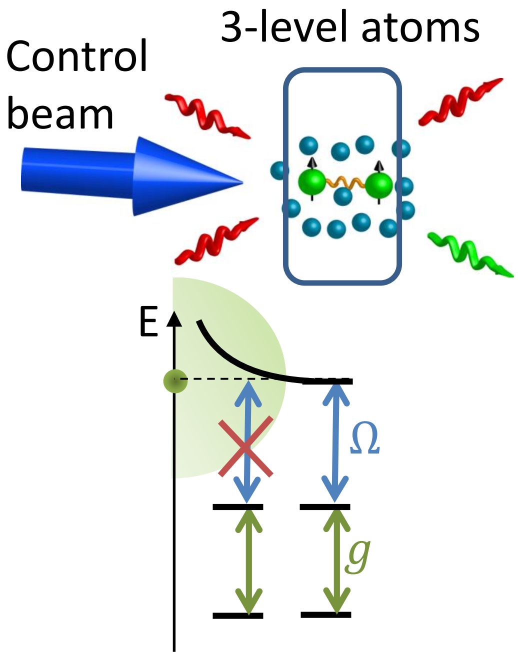
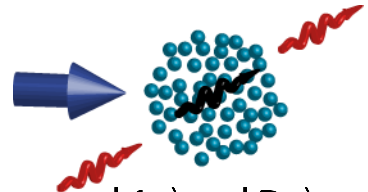
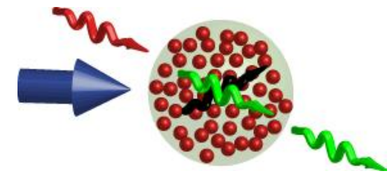
Interactions V_{vdW}	$\propto {n^*}^{11}/R^6$	100 **MHz** @ 10 **µm**

Rydberg Atoms





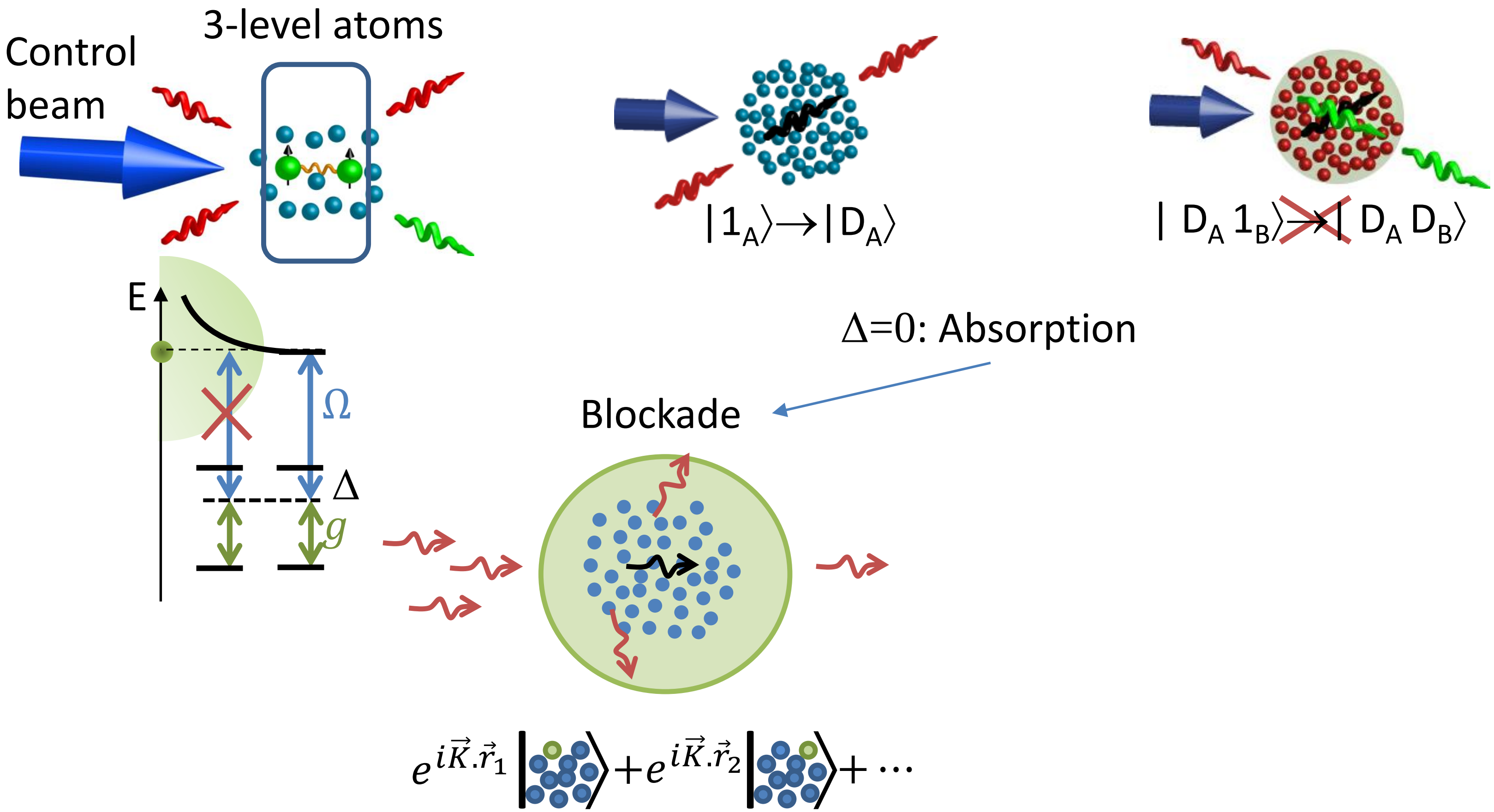
$|1_A\rangle \rightarrow |D_A\rangle$



$|D_A 1_B\rangle \not\rightarrow |D_A D_B\rangle$

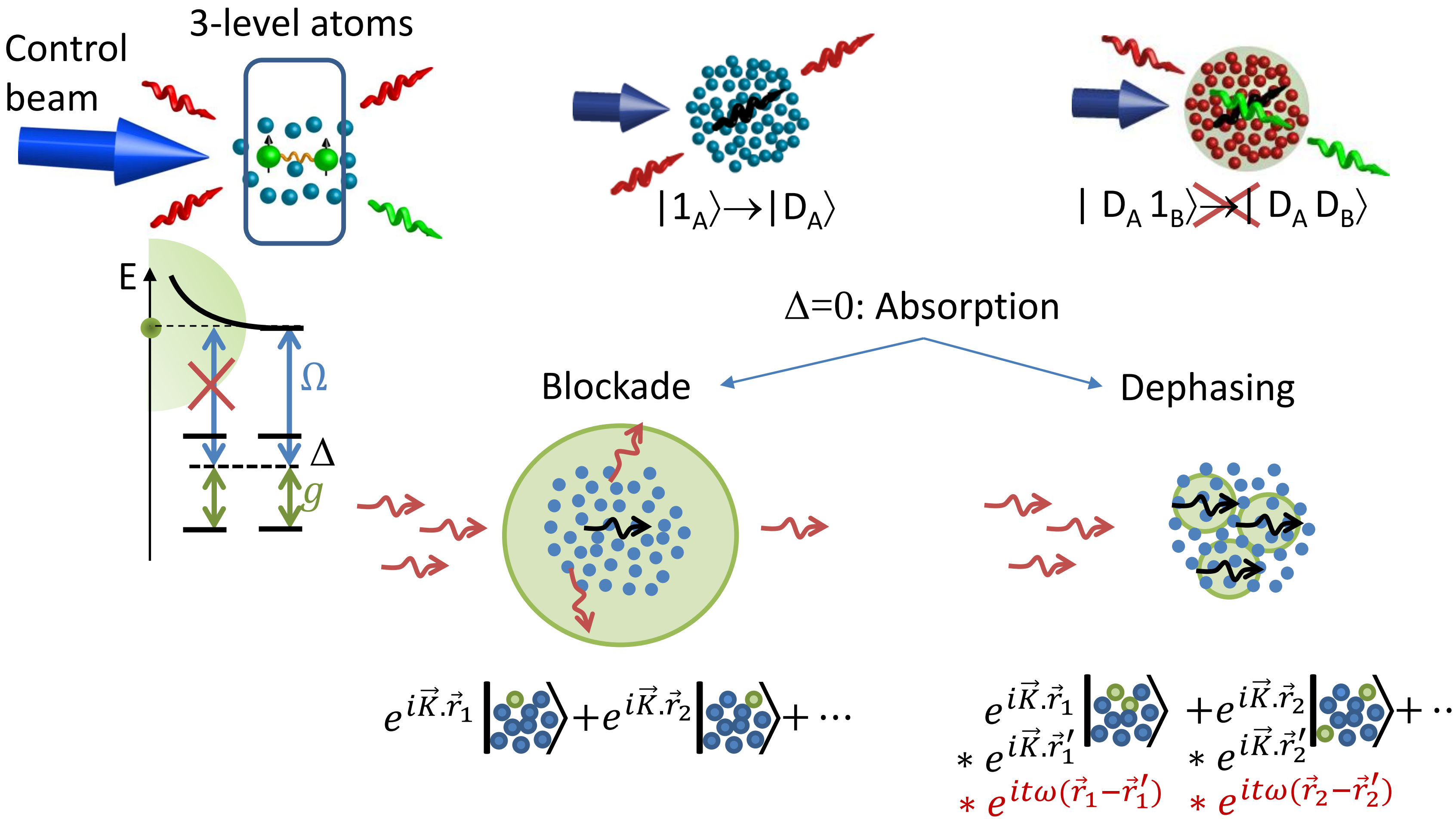
$$|D\rangle = \cos(\beta)\,|G,1\rangle - \sin(\beta)\,|R,0\rangle$$

Rydberg Atoms



Dudin & Kuzmich, Science **336,** 887 (2012)
Peyronel *et al,* Nature **488**, 57 (2012)
Baur *et al*, PRL **112**, 073901 (2014)
Gorniaczyk *et al,* PRL **113**, 053601 (2014)

Rydberg Atoms



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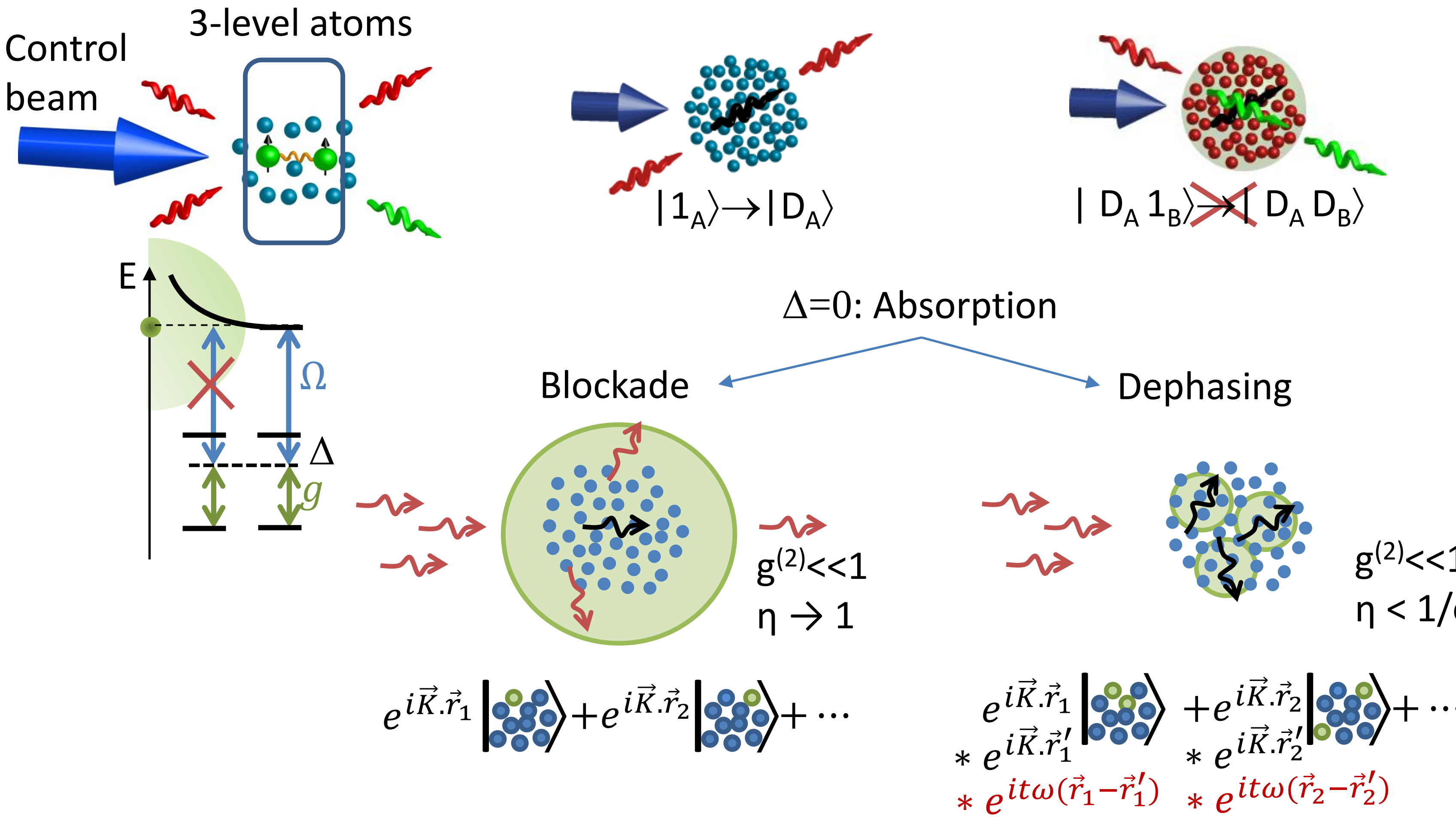
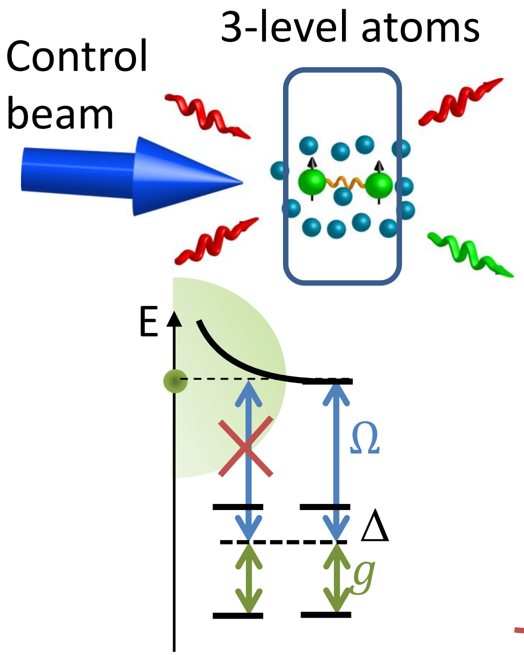
Rydberg Atoms

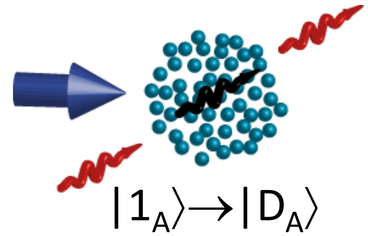


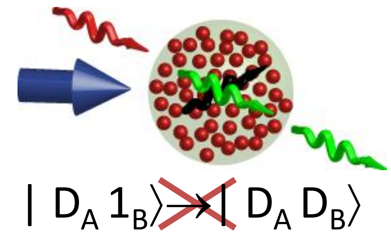
Dudin & Kuzmich, Science **336,** 887 (2012)
Peyronel *et al,* Nature **488**, 57 (2012)
Baur *et al*, PRL **112**, 073901 (2014)
Gorniaczyk *et al,* PRL **113**, 053601 (2014)

⇒ Single photon sources / transistors

Rydberg Atoms

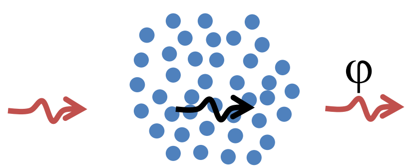


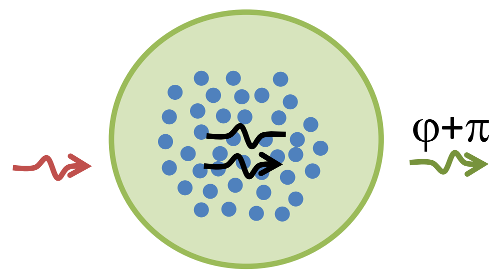




$\Delta \neq 0$: Dispersion

$\Rightarrow$ Two-photon gate



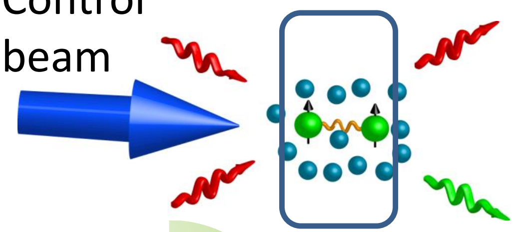


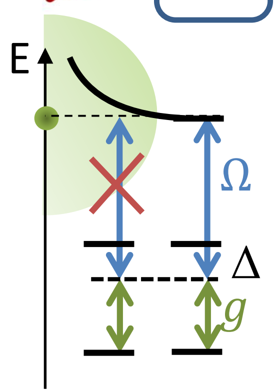
Tiarks &al,
Nat. Phys. **15**,
124 (2019)

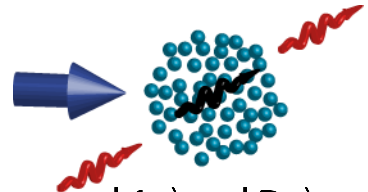
Rydberg Atoms

Control beam

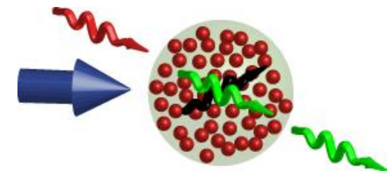
3-level atoms







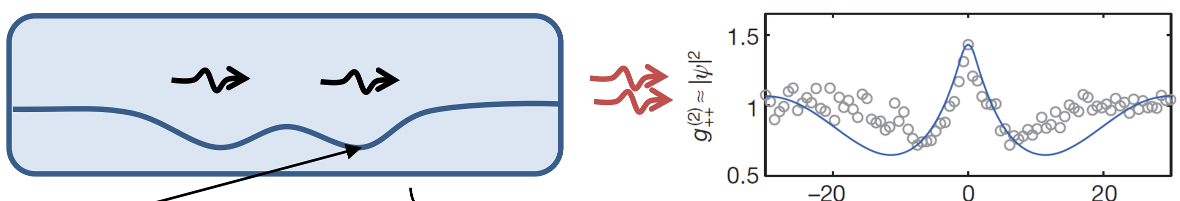
$|1_A\rangle \rightarrow |D_A\rangle$



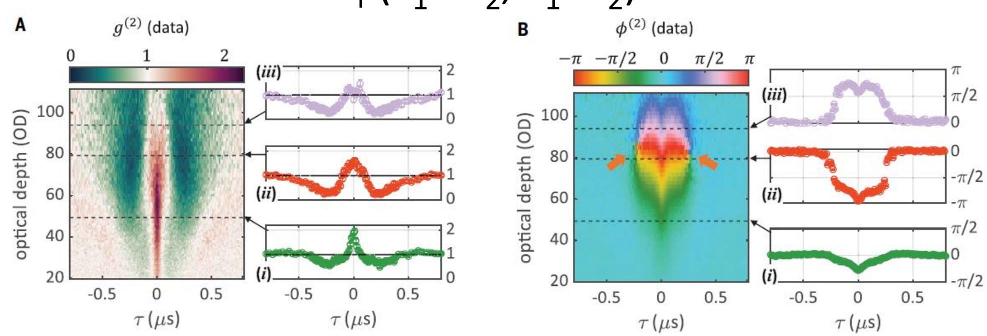
$| D_A\, 1_B\rangle \cancel{\rightarrow} | D_A\, D_B\rangle$

Δ≠0: Dispersion

⇒ Photon molecules / vortices



$\psi(x_1 + x_2,\, x_1 - x_2)$

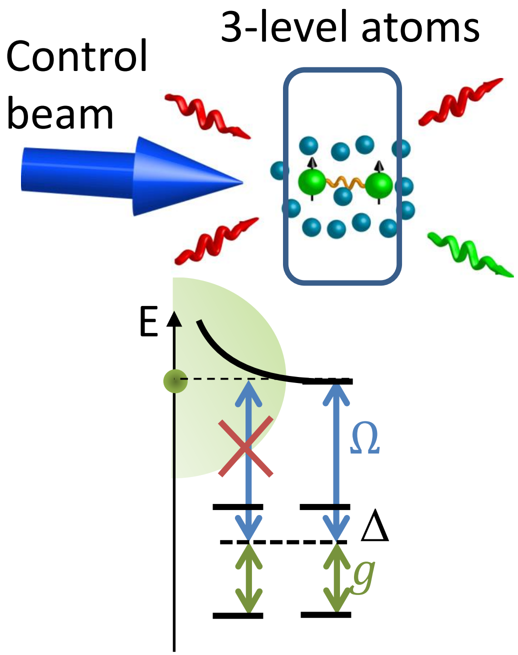


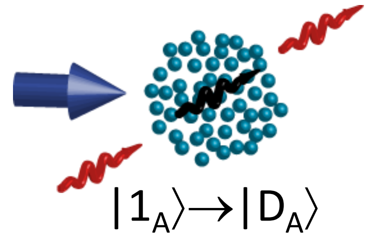
Polariton A
changes the optical index
seen by B
⇒ Group velocity dispersion
⇒ attraction

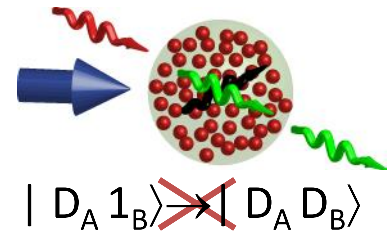
Firstenberg &al, Nature **502**, 71 (2013)

Drori &al, Science **381**, 193 (2023)

Rydberg Atoms

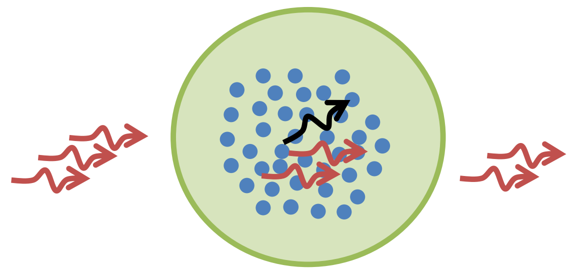


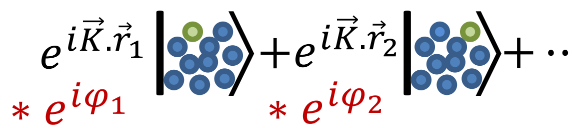




$\Delta \neq 0$: Dispersion

$\Rightarrow$ Single photon subtraction

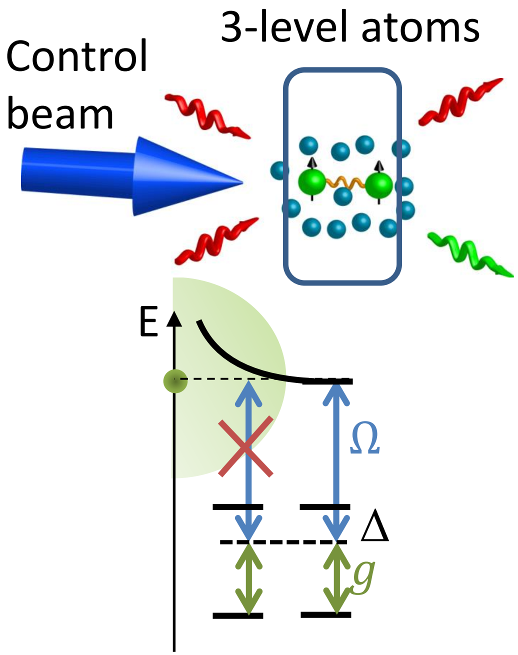


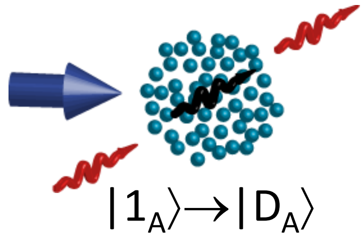


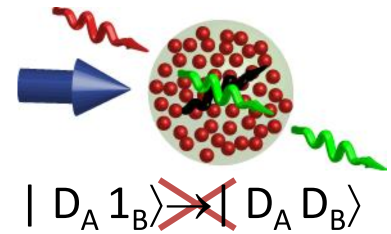
Inhomogeneous dephasing:
Rydberg polariton « trapped », other photons transmitted

Tresp &al, PRL. **117**, 223001 (2016)

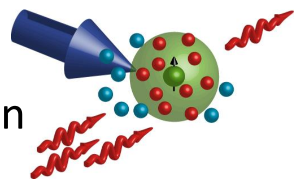
Rydberg Atoms



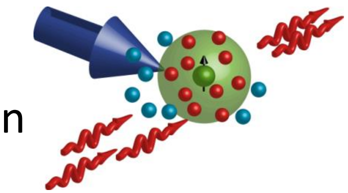




Δ=0:
Absorption



Δ≠0:
Dispersion



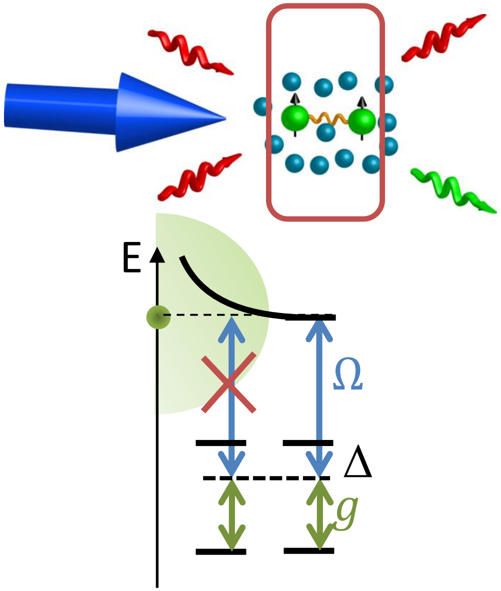
Photon sources,
transistors,

Photonic molecules/vortices,
2-photon gate, photon subtraction

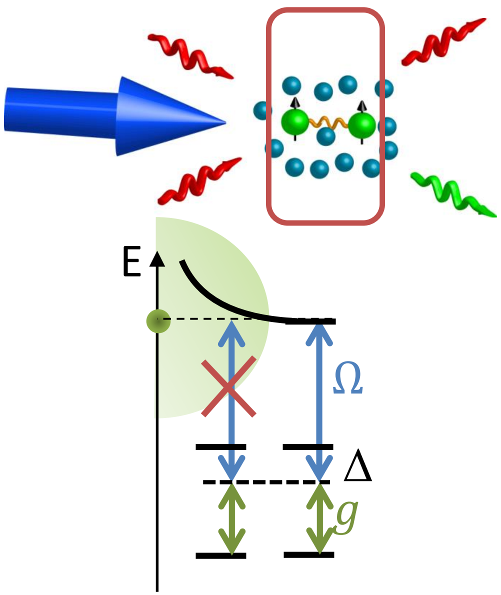
Reviews:

Murray & Pohl, AAMOP **65**, 321 (2016)
Firstenberg, Adams & Hofferberth, J. Phys. B **49,** 152003 (2016)
Wu &al, Chin. Phys. B **30**, 020305 (2021)
Shao &al, Appl. Phys. Rev. **11**, 031320 (2024).

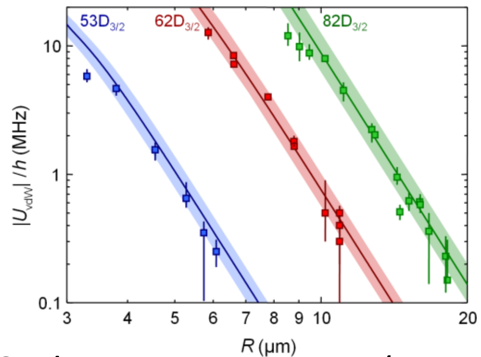
Rydberg Atoms



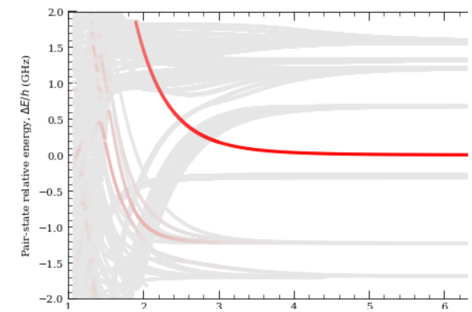
Rydberg Atoms



Short range $V \propto \cancel{C_6/r^6}$

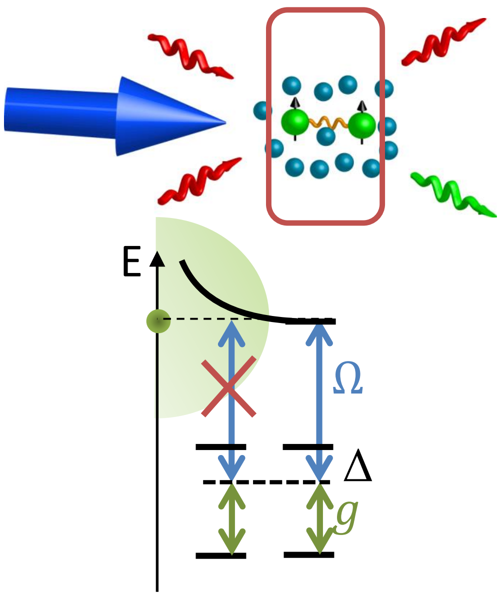


Béguin & al, PRL **110**, 263201 (2013)



atomcalc.jqc.org.uk

Rydberg Atoms



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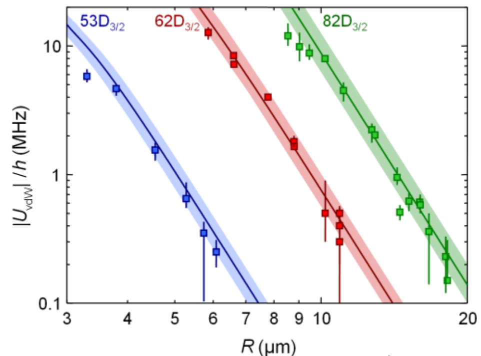
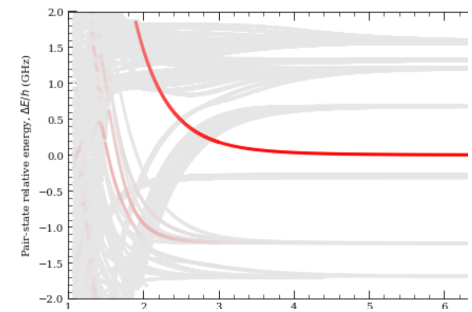
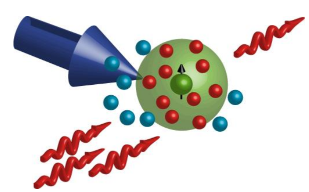


Béguin & al, PRL **110**, 263201 (2013)

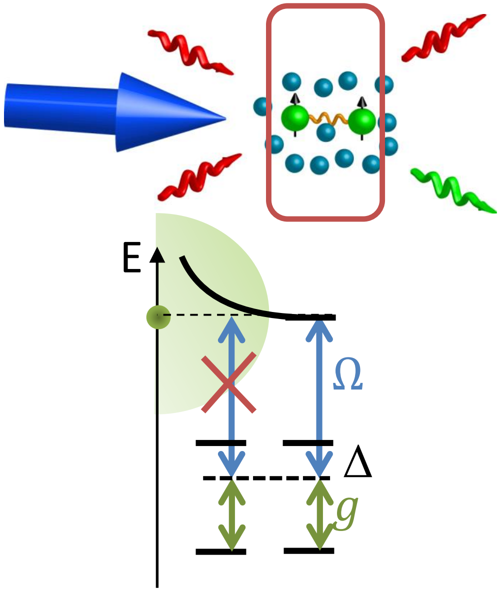


atomcalc.jqc.org.uk

Δ=0:



Rydberg Atoms



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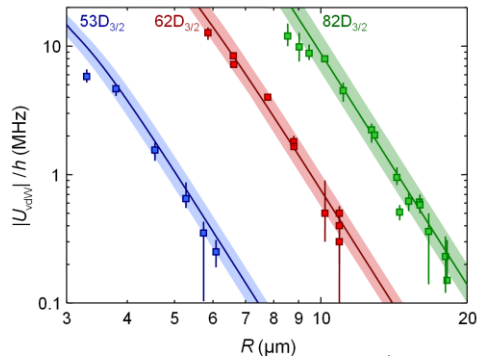
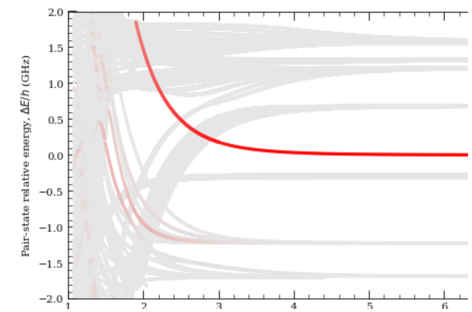
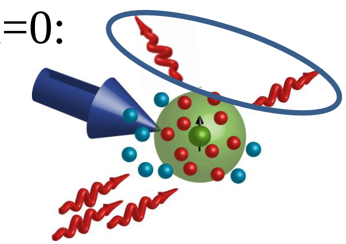




Béguin & al, PRL **110**, 263201 (2013)

atomcalc.jqc.org.uk

Δ=0:

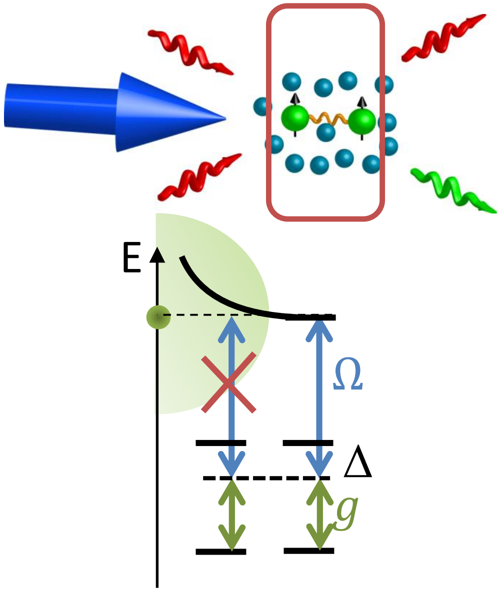


Transmitted & scattered photons are spatially entangled

$\Rightarrow$ Purity<1

Gorshkov &al, PRL **110**, 153601 (2013)

Rydberg Atoms



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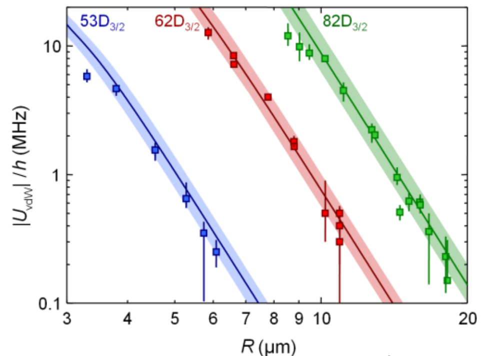
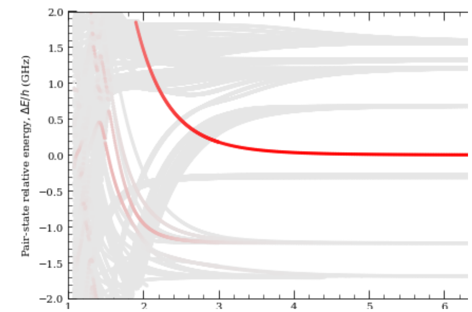
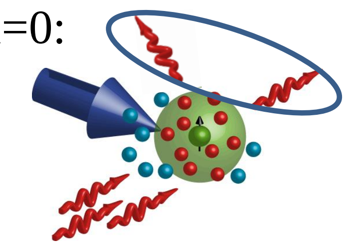


Béguin & al, PRL **110**, 263201 (2013)



atomcalc.jqc.org.uk

$\Delta=0$:



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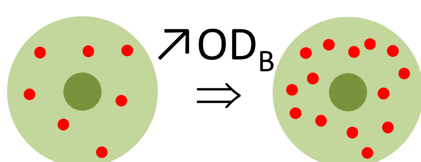
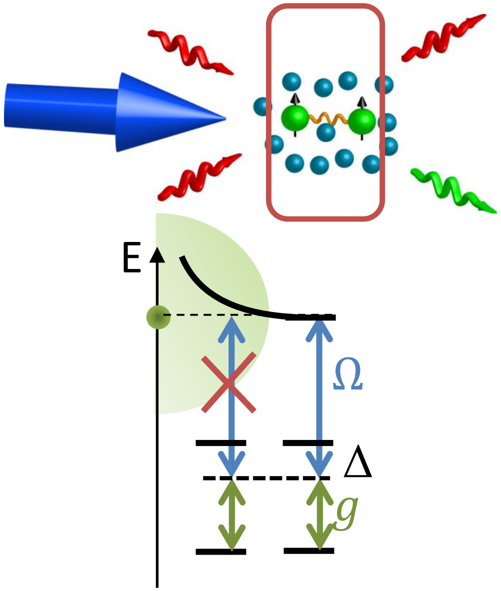
Gorshkov &al, PRL **110**, 153601 (2013)

$\Delta\neq0$: Dispersion $\propto OD_B/\Delta$,

Absorption $\propto OD_B/\Delta^2$



Rydberg Atoms



Short range

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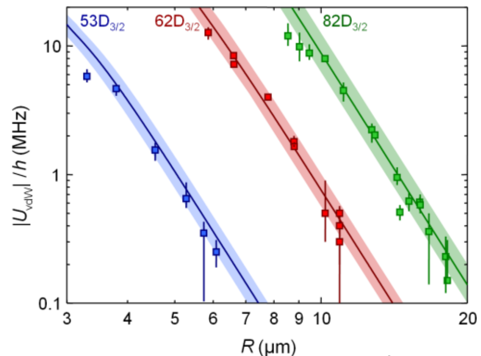
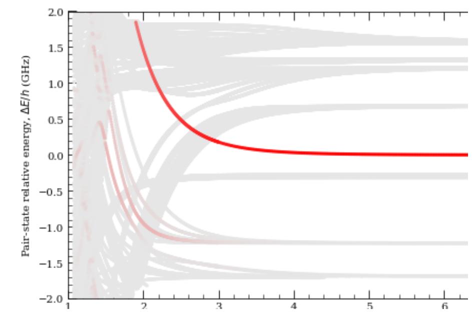


Béguin & al, PRL **110**, 263201 (2013)



atomcalc.jqc.org.uk

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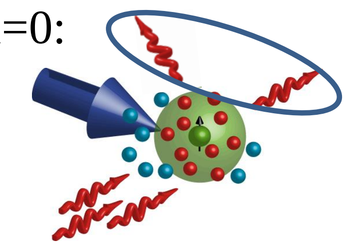


Transmitted & scattered photons are spatially entangled

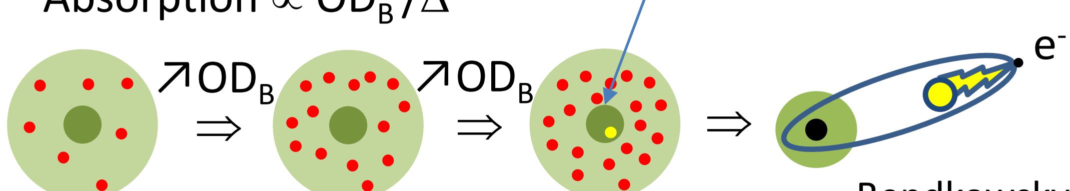
$\Rightarrow$ Purity<1

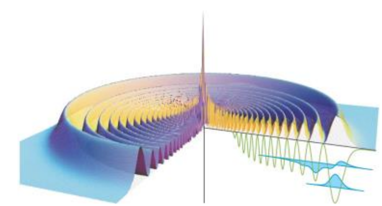
Gorshkov &al, PRL **110**, 153601 (2013)

$\Delta\neq0$: Dispersion $\propto OD_B/\Delta$,

Absorption $\propto OD_B/\Delta^2$

$\langle r\rangle \propto a_0 n^{*2} \sim 1$ μm!

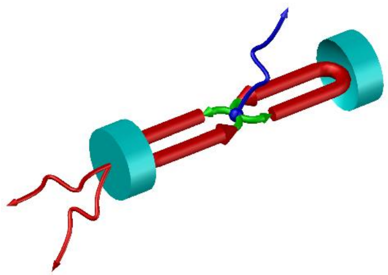




Bendkowsky &al, Nature **458**, 1005 (2009)

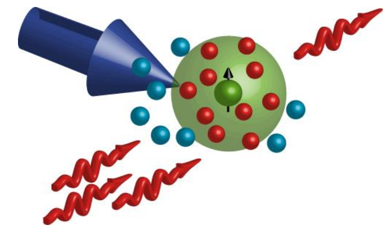
Rydberg superatom in a cavity?

Single atoms in cavities



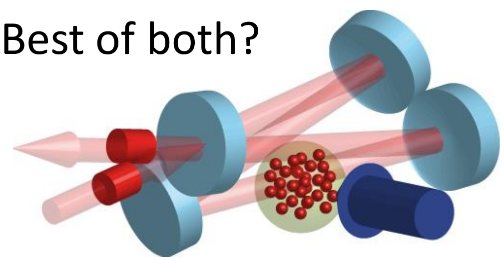
« Technical » issues

Rydberg ensembles in free space



« Physical » issues

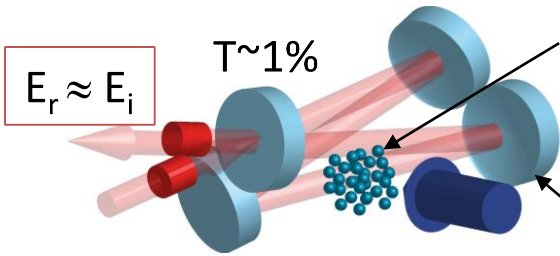
Best of both?



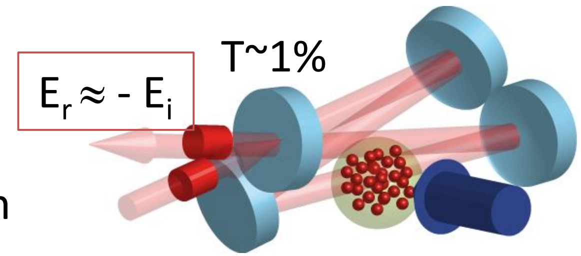
$$\boxed{C = N C_0}$$

Cooperativity $C_0 = \frac{g^2}{2(\kappa_0 + \kappa_\perp)\gamma} = \frac{1}{Transmission + Losses} * \frac{\sigma_{Atom} \sim \lambda^2 \nearrow ?}{\sigma_{Photon} \sim w^2 \searrow}$

Rydberg superatom in a cavity



$|G\rangle$: Losses <<T
Optical over-coupling

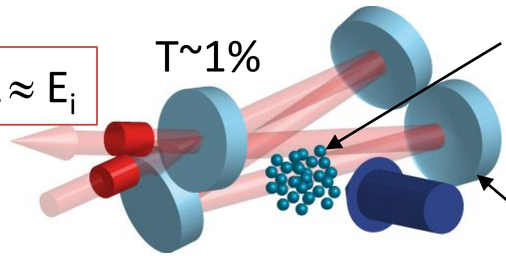


$|R\rangle$: Losses >>T
Optical under-coupling

Rydberg superatom in a cavity

$E_r \approx E_i$

T~1%



~800 Rb atoms

σ=5µm, T=3µK

Length~4x21mm

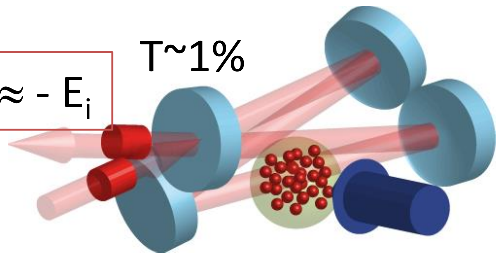
Finesse=590

|G⟩: Losses <<T

Optical over-coupling

$E_r \approx - E_i$

T~1%



|R⟩: Losses >>T

Optical under-coupling

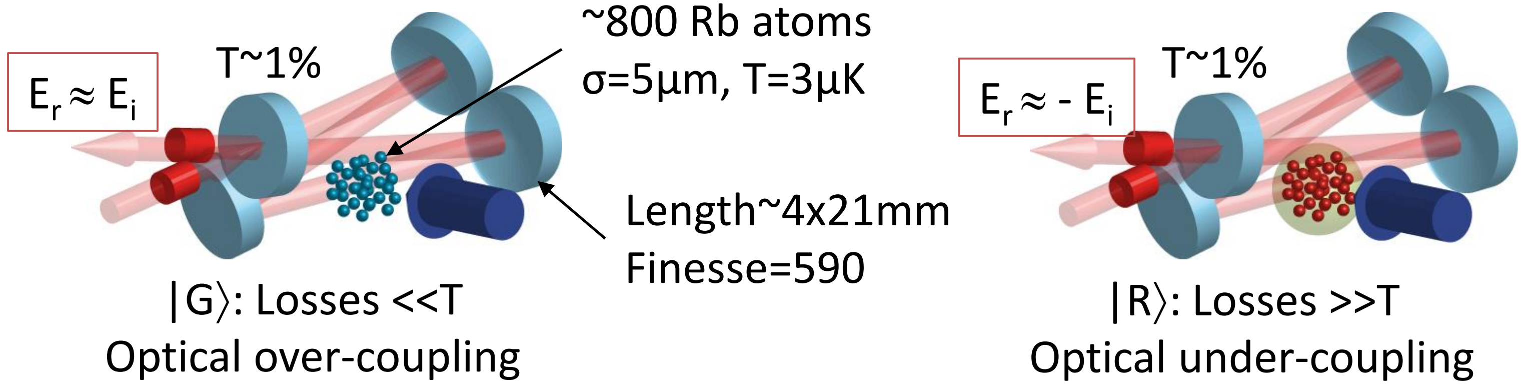
- Large-volume, medium-finesse cavity with non-trivial geometry

Easier to trap & cool atoms

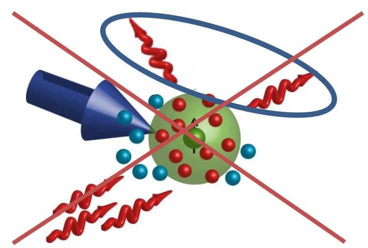
Easier to fabricate

More interesting physics

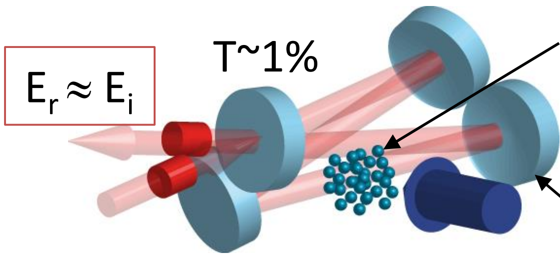
Rydberg superatom in a cavity



- Large-volume, medium-finesse cavity with non-trivial geometry
- Free-space scattering suppressed



Rydberg superatom in a cavity



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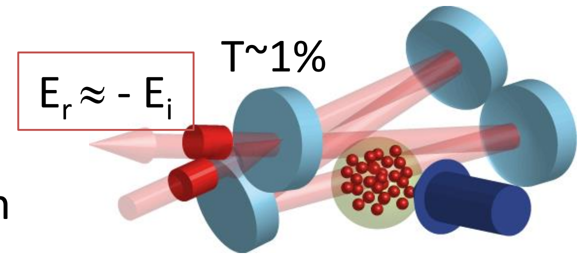

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- Large-volume, medium-finesse cavity with non-trivial geometry
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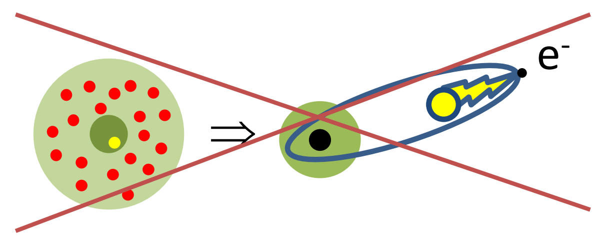
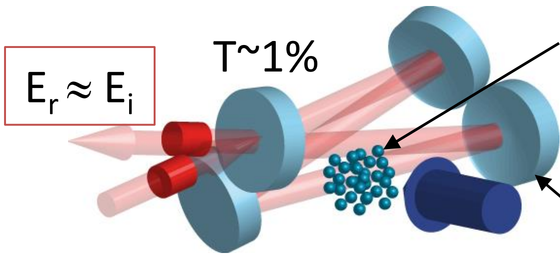
- L compares to T~1% << 1: moderate density OK



Rydberg superatom in a cavity



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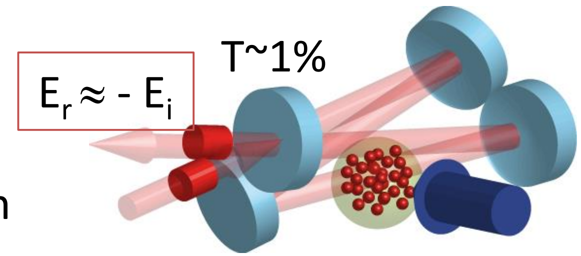
Optical over-coupling



$|R\rangle$: Losses >>T
Optical under-coupling

- Large-volume, medium-finesse cavity with non-trivial geometry
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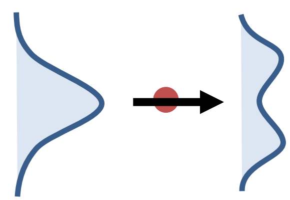
- L compares to T~1% << 1: moderate density OK
- Cloud can be smaller than mode



Rydberg superatom in a cavity

$\boxed{E_r \approx E_i}$ T~1%

$$\Delta \gg \tilde{g}$$

~800 Rb atoms
σ=5μm, T=3μK

Length~4x21mm
Finesse=590

|G⟩: Losses <<T
Optical over-coupling

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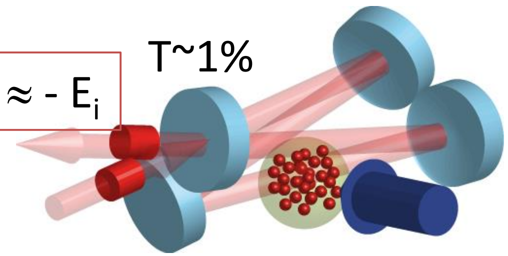

|R⟩: Losses >>T
Optical under-coupling

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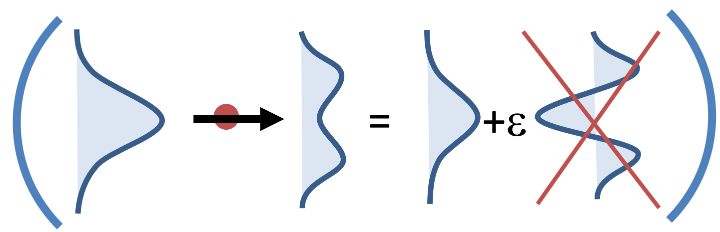
- Free-space scattering suppressed
- L compares to T~1% << 1: moderate density OK
- Cloud can be smaller than mode



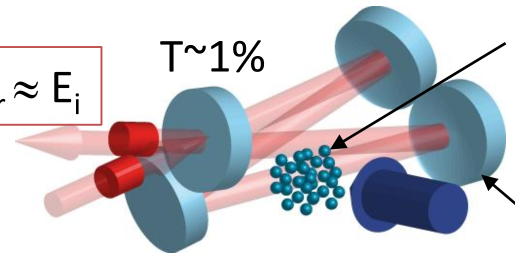
$$\begin{pmatrix} 0 & \tilde{g} \\ \tilde{g} & \Delta \end{pmatrix}$$

$\Delta \gg \tilde{g}$

Rydberg superatom in a cavity

$E_r \approx E_i$

T~1%



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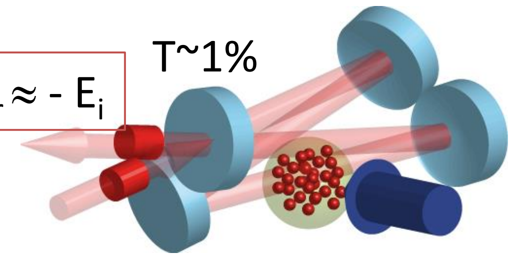
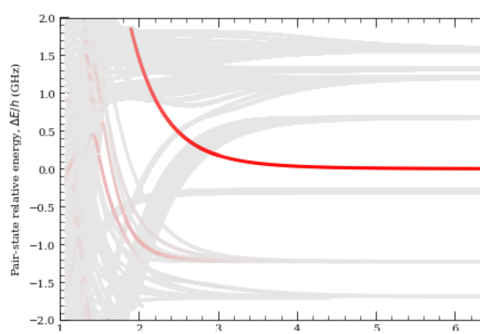
T~1%

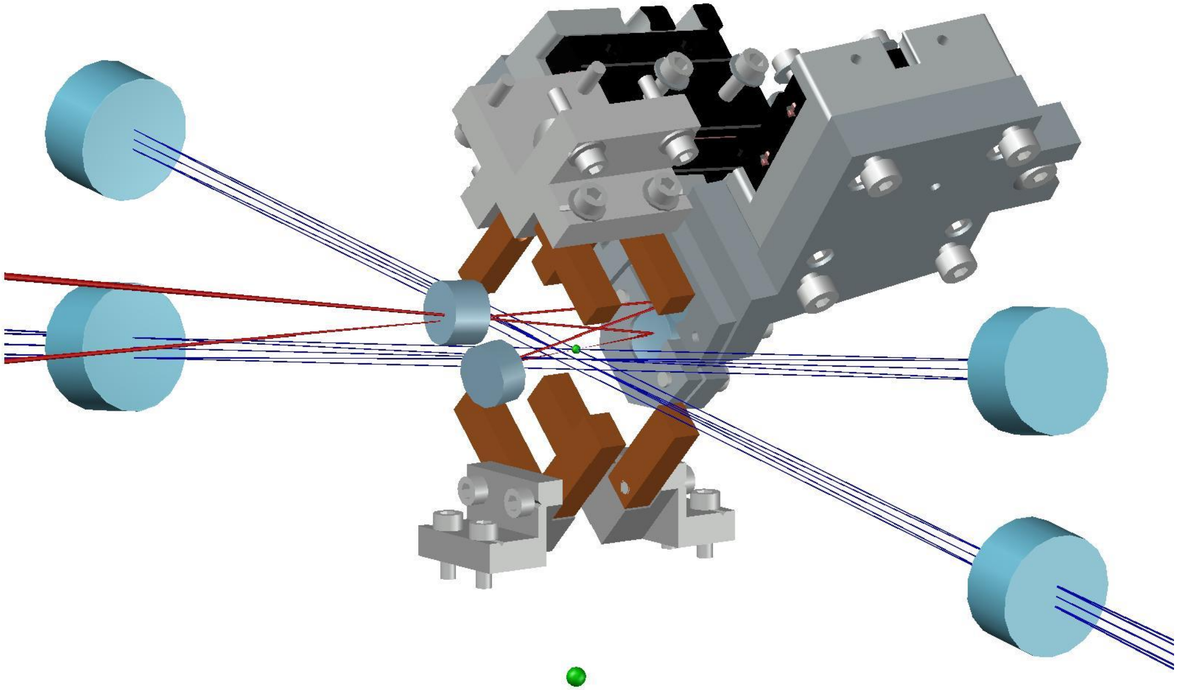


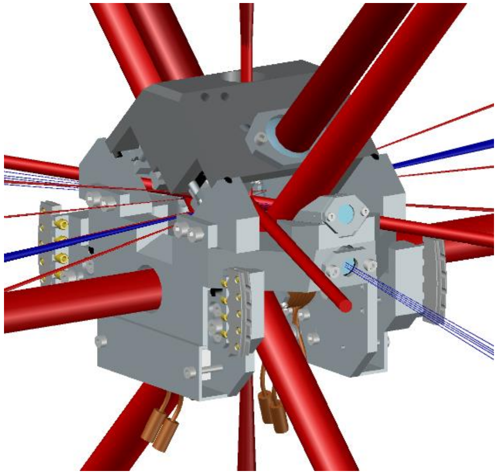
$|R\rangle$: Losses >>T
Optical under-coupling

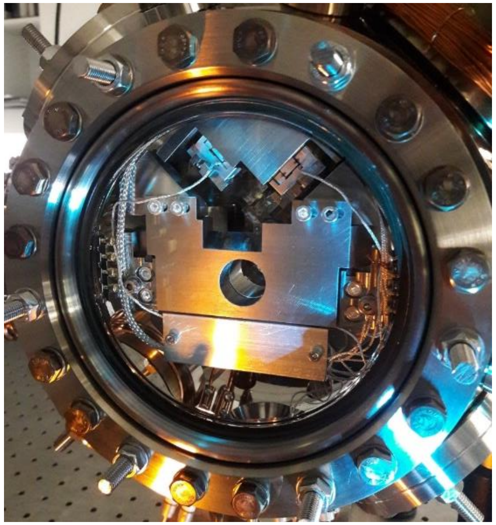
- Large-volume, medium-finesse cavity with non-trivial geometry
- Free-space scattering suppressed
- L compares to T~1% << 1: moderate density OK
- Cloud can be smaller than mode
- Details of interactions don't matter too much



Experimental setup

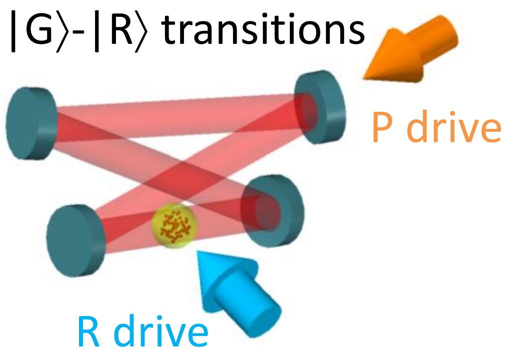


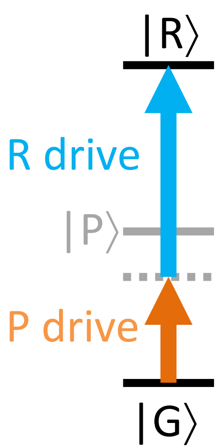


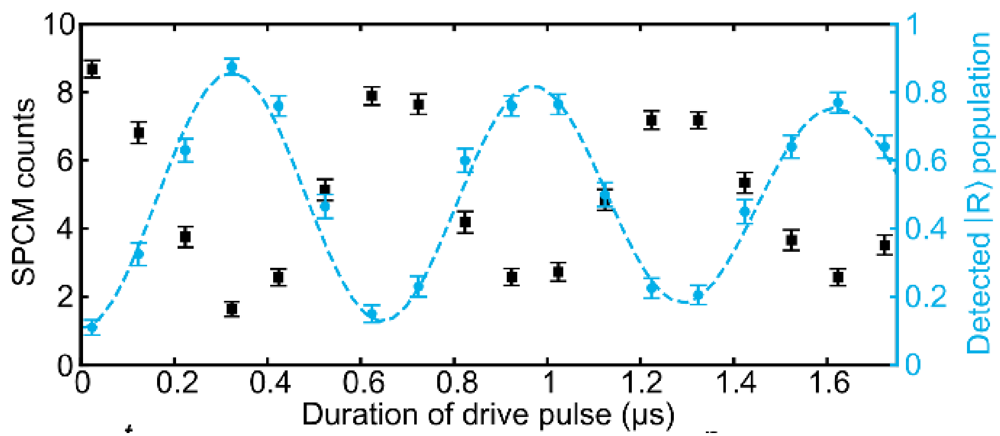


- Electric / magnetic field control
- Buildup cavities for Rydberg lasers
- Flexible cavity geometry
- High resolution optics
- ~10 Hz total repetition rate, x10 cloud recycling

Superatom state control & detection





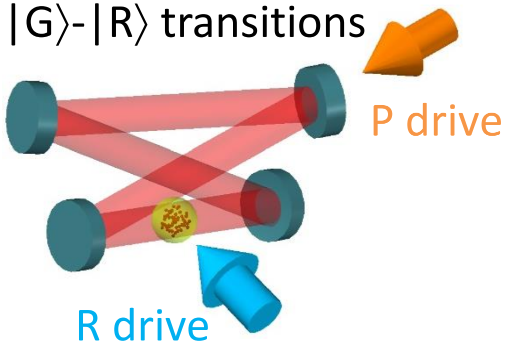


|G⟩ = |$5S_{1/2}$, F=1, m_F=1 ⟩
|P⟩ = |$5P_{3/2}$, F=2, m_F=2 ⟩
|R⟩ = |$109S_{1/2}$, J=1/2, m_J=1/2 ⟩

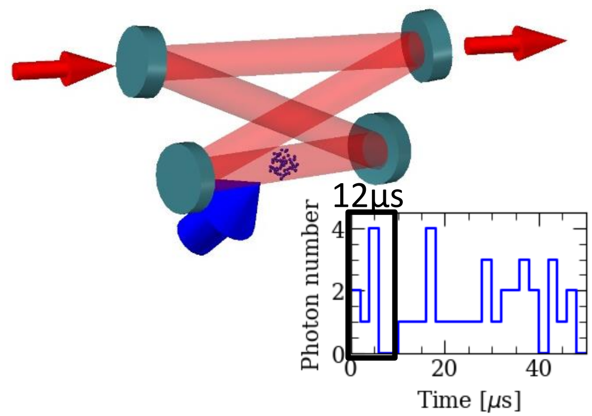
Vaneecloo &al,
PRX **12**, 021034 (2022)

Superatom state control & detection

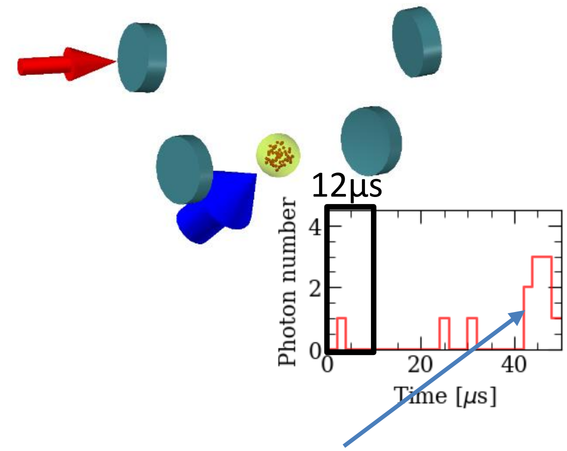
|G⟩-|R⟩ transitions



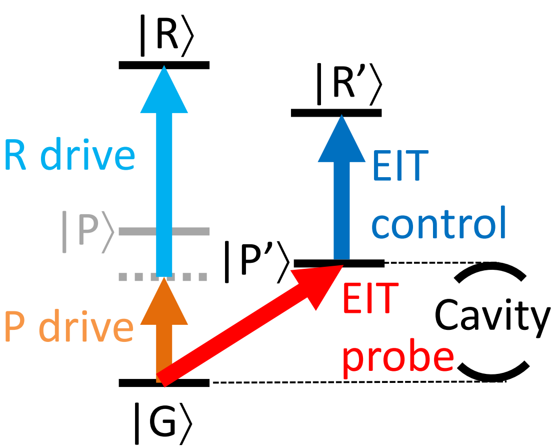
Detection in|G⟩

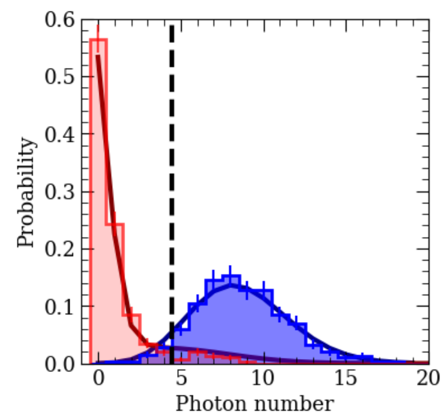


Detection in|R⟩



Quantum jump to |G⟩:
Photon flux x 20





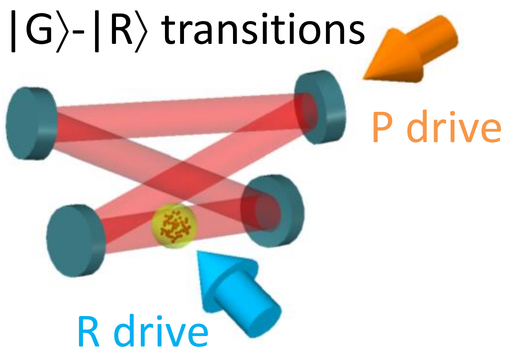
Single-shot detection efficiency 95% in 12µs:

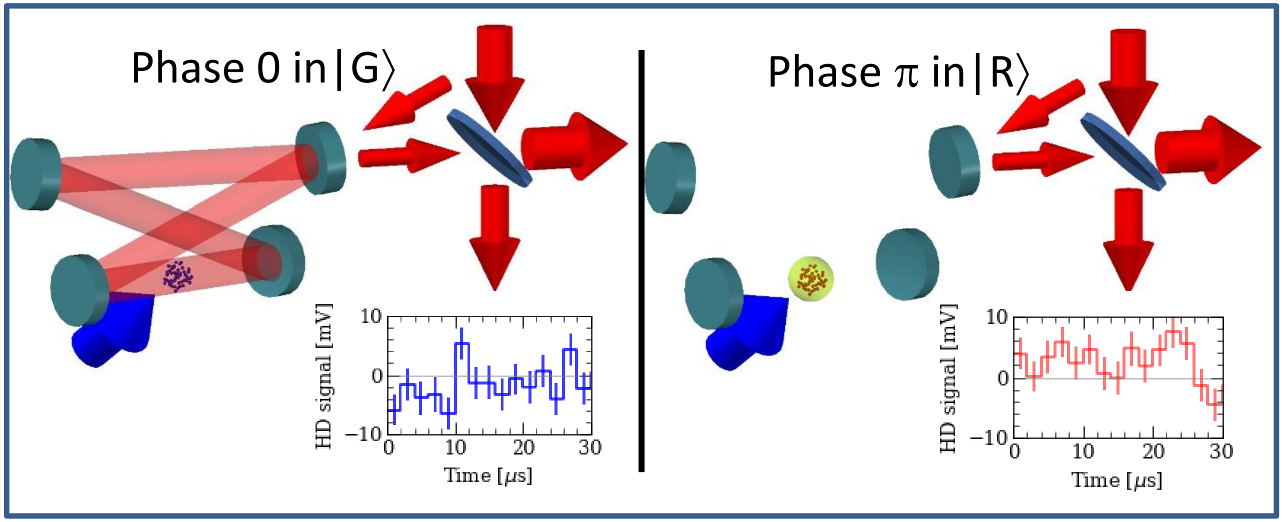
$|G\rangle = |5S_{1/2}, F=1, m_F=1\rangle$
$|P\rangle = |5P_{3/2}, F=2, m_F=2\rangle$
$|R\rangle = |109S_{1/2}, J=1/2, m_J=1/2\rangle$

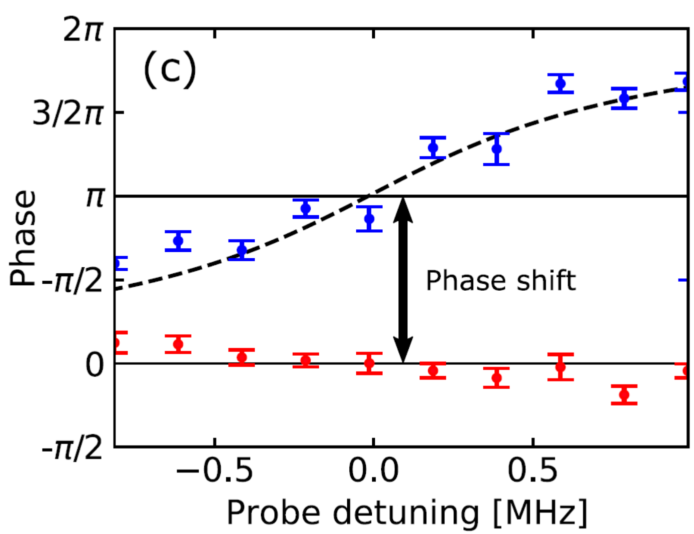
$|P'\rangle = |5P_{1/2}, F=2, m_F=2\rangle$
$|R'\rangle = |78S_{1/2}, J=1/2, m_J=1/2\rangle$

Vaneecloo &al,
PRX **12**, 021034 (2022)

Controlled optical π phase shift



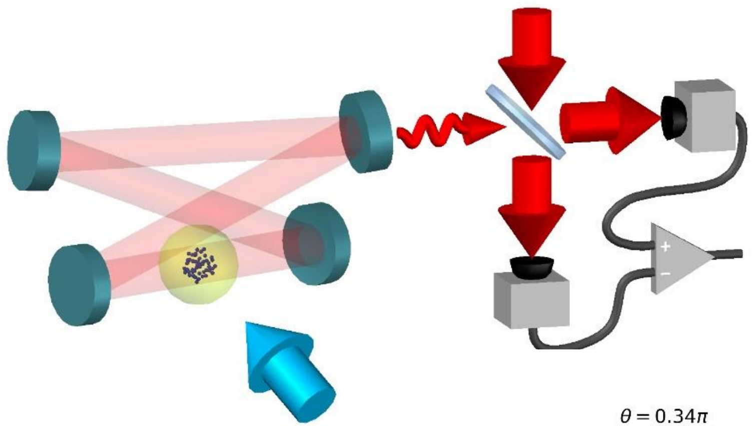




C-Z gate: Stolz et al, PRX **12**, 021035 (2022)

Vaneecloo &al, PRX **12**, 021034 (2022)

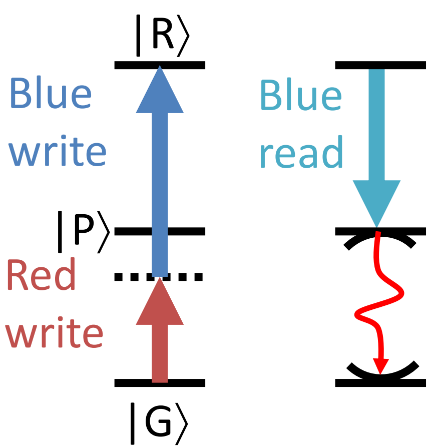
Wigner-negative photonic qubits

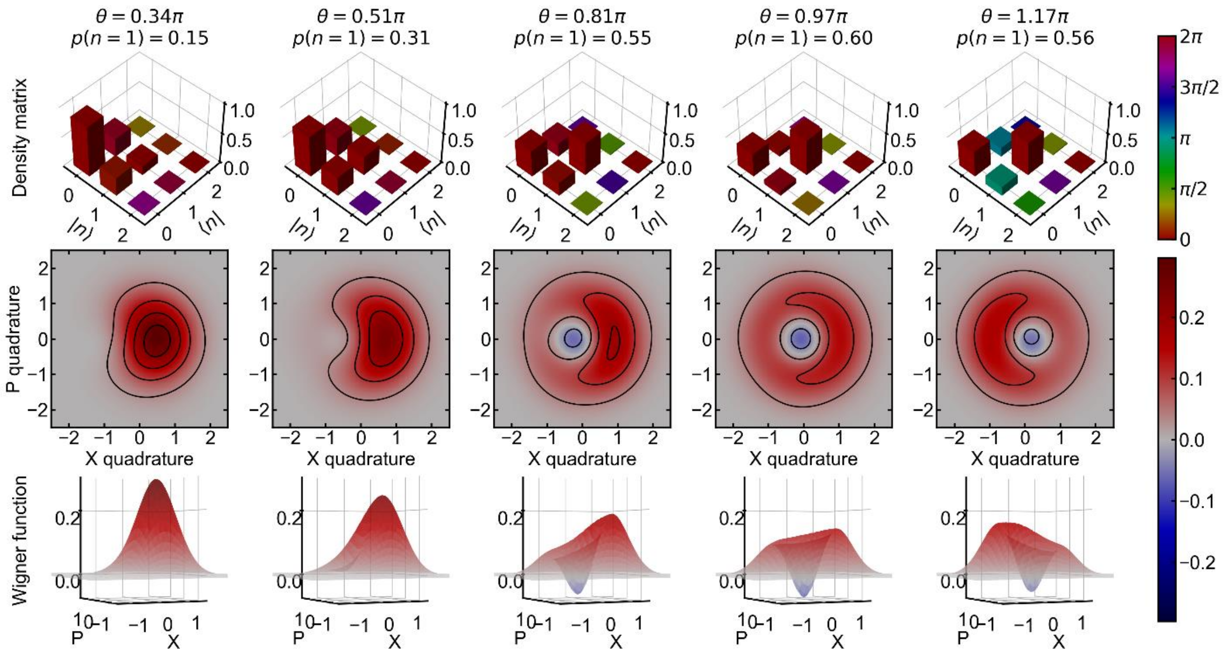


$$\cos\frac{\theta}{2}|G\rangle - \sin\frac{\theta}{2}|R\rangle$$

$$|D\rangle = \cos(\beta)|G,1\rangle - \sin(\beta)|R,0\rangle$$

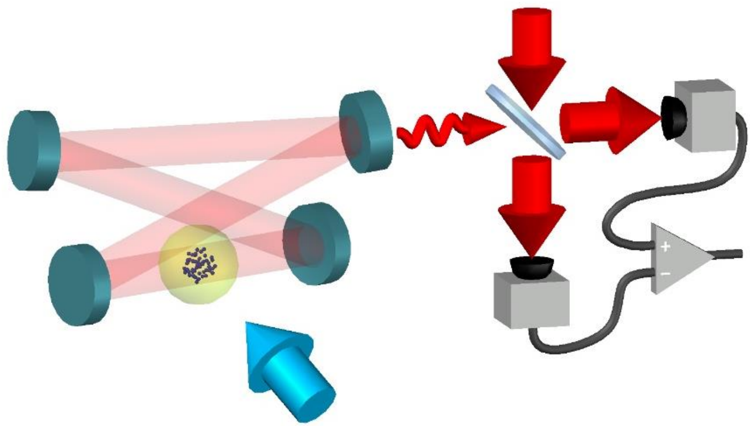
$$\boxed{\cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}|1\rangle}$$





Magro &al, Nature Photonics **17**, 688 (2023)

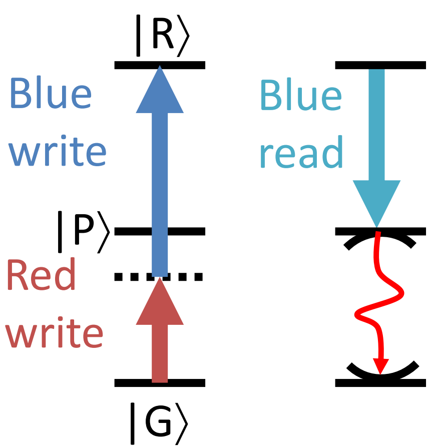
Wigner-negative photonic qubits

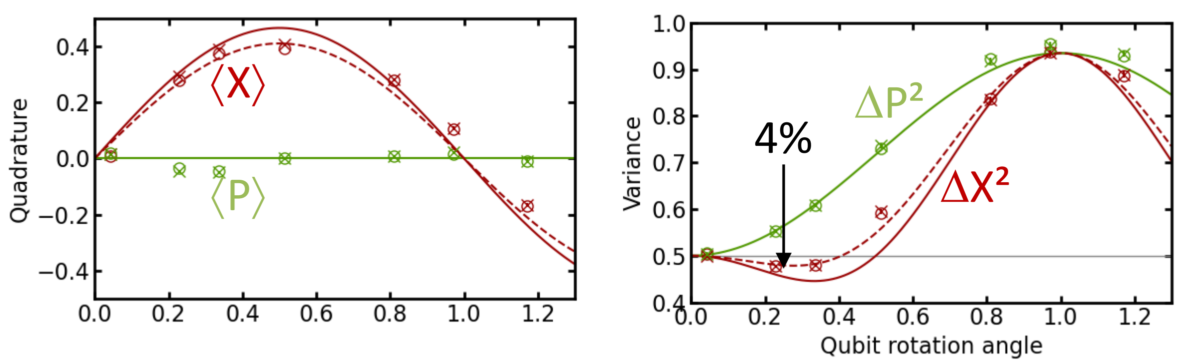


$$|\psi(0)\rangle = \cos\left(\frac{\theta}{2}\right)|G\rangle - \sin\left(\frac{\theta}{2}\right)|R\rangle$$

Quadrature squeezing:

	ΔX^2	ΔP^2	θ_{opt}
Decay $\gamma_\parallel$	↘	↘	$\pi/3$
Dephasing $\gamma_\perp$	↘	→	↘

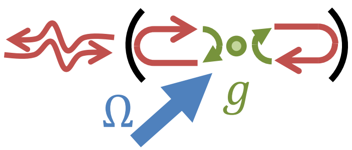


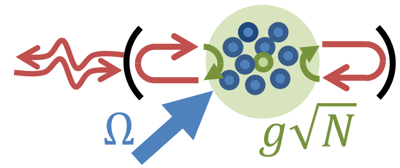


$\Rightarrow \gamma_\parallel$ & $\gamma_\perp$: alternative for Ramsey spectroscopy?

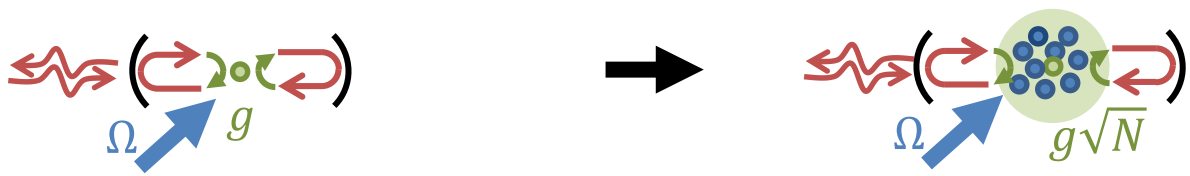
Magro &al, Nature Photonics **17**, 688 (2023)

Understanding superatoms





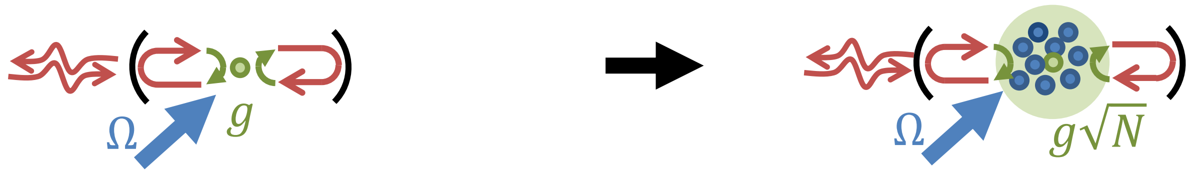
Understanding superatoms



Qubits must be:

Identical:
Fluctuations of N_{atoms}?

Understanding superatoms

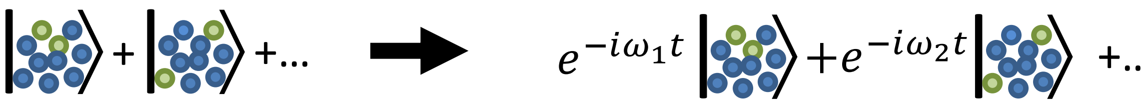


Qubits must be:

Identical:
Fluctuations of N_{atoms}?

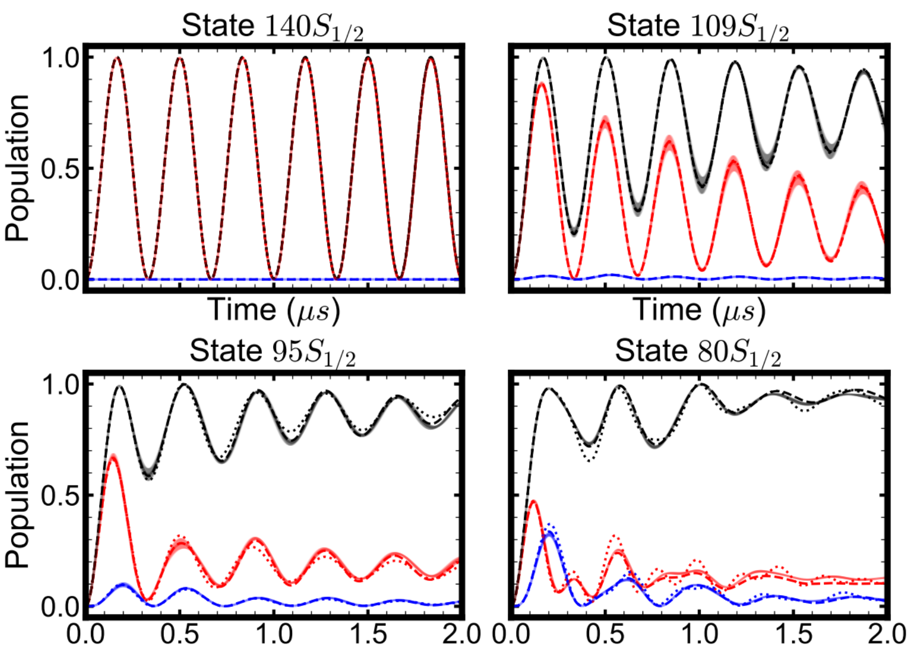
Anharmonic:
Imperfect blockade?

Imperfect blockade

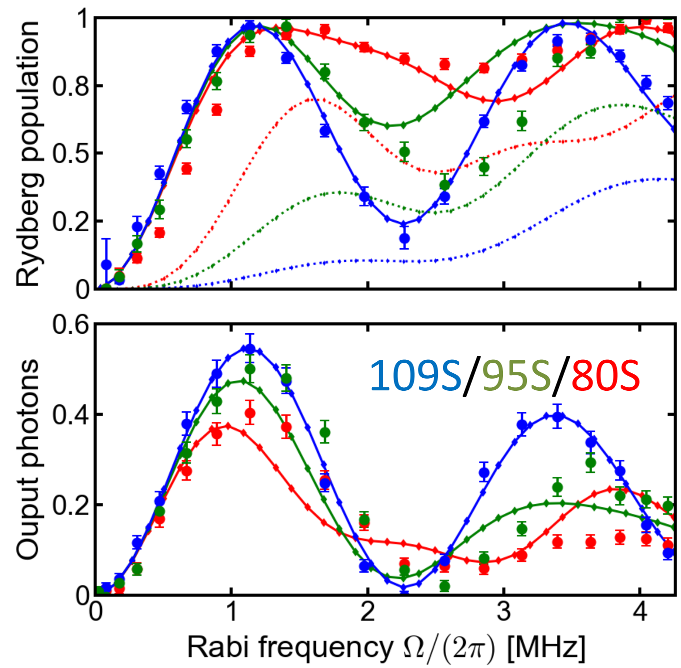


Low-dimensional parameter-free model, from first priciples:

Vs brute-force numerics:

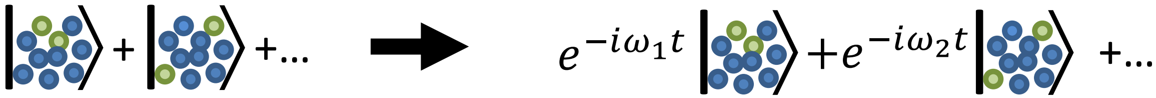


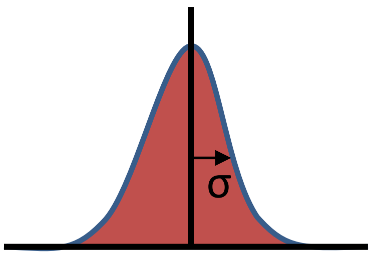
Vs data:



Magro et al, in preparation

Imperfect blockade

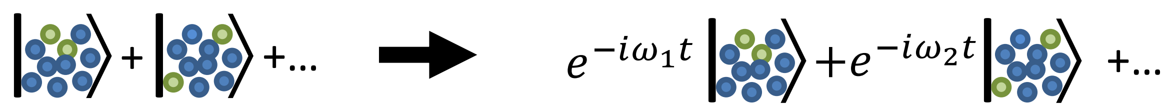


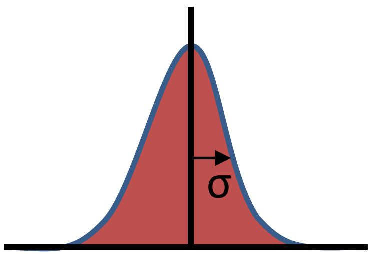


- $V_0=C_6/\sigma^6$~10 GHz
- Expt: V_{eff}~1 MHz…?

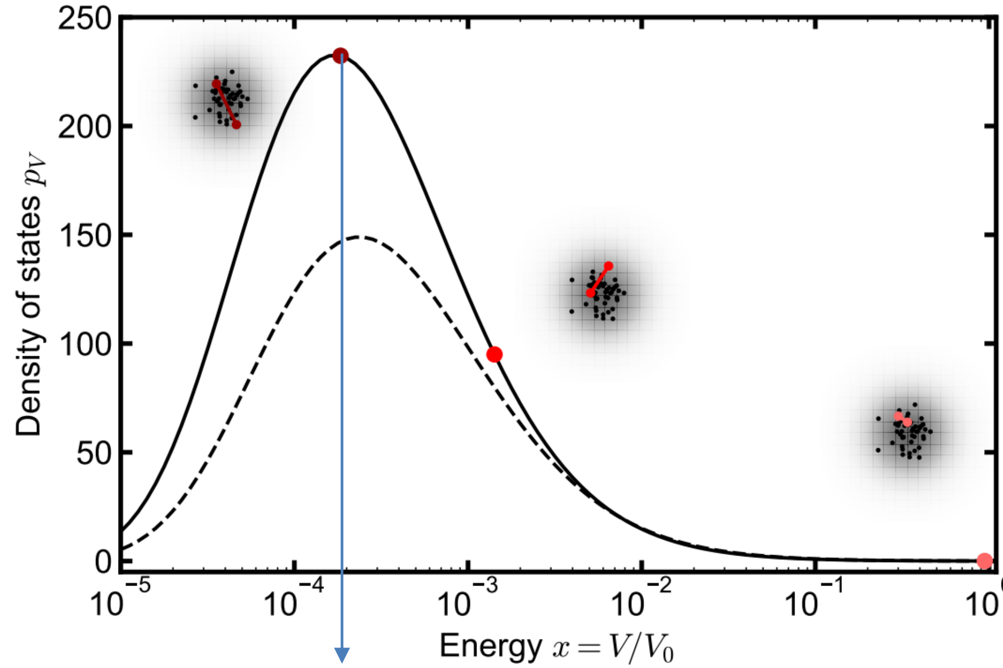
Magro et al, in preparation

Imperfect blockade





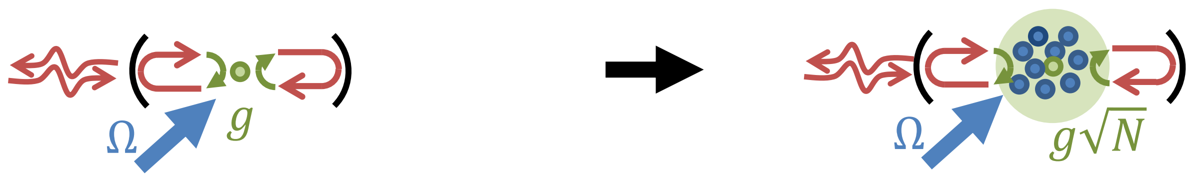
- $V_0 = C_6/\sigma^6 \sim 10$ GHz
- Expt: $V_{eff} \sim 1$ MHz…?



$$R_{eff} = \sigma * 3\sqrt{2}$$

Magro et al, in preparation

Understanding superatoms



Qubits must be:

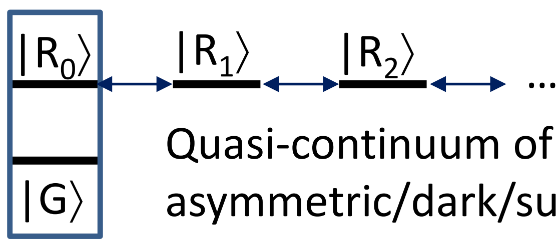
Identical:
Fluctuations of N_{atoms}?

Anharmonic:
Imperfect blockade?

Coherent:
Inhomogeneous dephasing?

$e^{-i\omega_1 t}$  $+ e^{-i\omega_2 t}$  $+ \cdots$

$\Rightarrow$

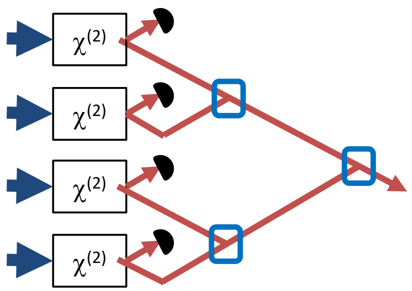


Quasi-continuum of
asymmetric/dark/subradiant Dicke states

A. Covolo &al, Optica 12, 1427 (2025) : last talk of the day

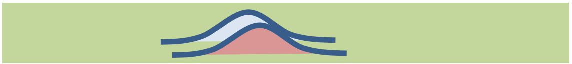
Conclusion

Projective measurements:



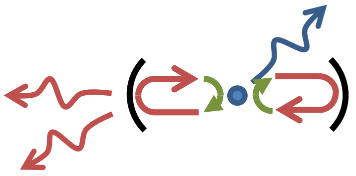
Strong industrial effort

Kerr-type non-linearities:



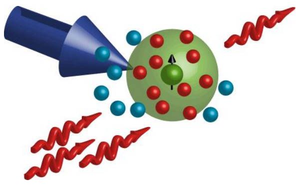
« What is proved by impossibility proofs is lack of imagination » (J.S. Bell)

Single emitters:



New cavities

Rydberg atoms:



+ Cavities / waveguides